Sticky Leverage

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Introduction

Models of monetary non-neutrality have traditionally emphasized the importance of sticky prices and/or wages

- This seems perhaps overdone

We focus on an alternative channel for monetary non-neutrality
Nominal debt that is both long-term and defaultable

- This is both large and quite costly to adjust (at least the principal)

This creates two problems for firms

- Default risk
- Debt overhang
Preview of Findings

- Debt deflation is a quantitatively powerful propagation mechanism
- Sticky or persistent leverage is the key
- Conventional Taylor rules can stabilize output in response to shocks
Related Literature

- No quantitative business cycle analysis with defaultable nominal, long-term debt
Model

Continuum of firms of measure one, firm $j$ produces

$$y_{jt}^j = A_t \left( k_t^j \right)^\alpha \left( n_t^j \right)^{1-\alpha}$$

with aggregate productivity $\ln A_t = \rho \ln A_{t-1} + \sigma \varepsilon_t$, and

$$k_{t+1}^j = (1 - \delta + i_t^j) k_t^j \equiv g \left( i_t^j \right) k_t^j$$

Define

$$R_t k_t^j \equiv \max_{n_t^j} A_t \left( k_t^j \right)^\alpha \left( n_t^j \right)^{1-\alpha} - w_t n_t^j$$

After-tax operational profits, with idiosyncratic IID shock $z_t^j$

$$(1 - \tau) \left( R_t k_t^j - z_t^j k_t^j \right)$$
Debt

Nominal debt outstanding requires payment

\[(c + \lambda) \frac{b^j_t}{\mu_t}\]

where \(b_t \equiv B_t / P_{t-1}\), \(c\) coupon, \(\lambda\) amort., \(\mu_t\) inflation rate

After issuing \(s^j_t\)

\[b^j_{t+1} = (1 - \lambda) \frac{b^j_t}{\mu_t} + \frac{s^j_t}{p^j_t}\]

\(p^j_t\) market value of debt
Equity Value and Default

Value to equity holders/owners

\[ E \left( k_t^i, b_t^i, z_t^i, \mu_t \right) = \max \left[ 0, (1 - \tau) \left( R_t - z_t^i \right) k_t^i - \left( (1 - \tau) c + \lambda \right) \frac{b_t^i}{\mu_t} + V \left( k_t^i, b_t^i, \mu_t \right) \right] \]

where

\[ V \left( k_t^i, b_t^i, \mu_t \right) = \max_{b_{t+1}^i, k_{t+1}^i} \left\{ p_t^i \left( b_{t+1}^i - (1 - \lambda) \frac{b_t^i}{\mu_t} \right) - l_t^i + \tau \delta k_t^i + \right. \]

\[ E_t M_{t,t+1} E \left( k_t^i, b_t^i, z_t^i, \mu_t \right) \}

Firms default when

\[ (1 - \tau) \left( R_t k_t^i - z_t^i k_t^i \right) + V_t \left( k_t^i, b_t^i, \mu_t \right) < \left( (1 - \tau) c + \lambda \right) \frac{b_t^i}{\mu_t} \]
Debt Pricing

\[ b_{t+1}^j \rho_t^j = E_t M_{t,t+1} \left\{ \int_{z_{t+1}^j}^{z_{j,*}^j} \right. \]

\[ \Phi(z_{t+1}^j) \left[ c + \lambda \frac{b_{t+1}^j}{\mu_{t+1}} + (1 - \tau) \left( R_{t+1} k_{t+1}^j - z_{t+1}^j k_{t+1}^j \right) ight. \]

\[ + V \left( k_{t+1}^j, b_{t+1}^j, \mu_{t+1} \right) - \xi_k k_{t+1} + (1 - \lambda) \frac{\rho_{t+1}^j b_{t+1}^j}{\mu_{t+1}} \]

\[ \left. \left. \left. \left. \right. \right. \right. \right. \left. \right. \right. \right. \right. \right. \left. \right. \right. \right. \right. \right. \left. \right. \right. \left. \right. \right. \left. \right. \right. d\Phi(z_{t+1}^j) \right\} \]
Households and Equilibrium

Consumer/Investor preferences

$$\max_{\{C,N\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t [(1 - \theta) \ln C_t + \theta \ln (3 - N_t)]$$

Aggregate resource constraint

$$Y_t - [1 - \Phi(z^*)] \xi^r \xi K_t = C_t + I_t$$

Inflation Process

$$\ln \mu_t = (1 - \rho^\mu) \ln \mu + \rho^\mu \ln \mu_{t-1} + \epsilon^\mu_t$$
Characterization

\[ v(\omega, \mu) = \max_{\omega', i} \left\{ g(i) EM' \int_{z}^{z^*} \left( p \left( \omega' g(i) - (1 - \lambda) \frac{\omega}{\mu} \right) - i + \tau \delta + \right. \right. \]

\[ \left. \left. \left( 1 - \tau \right) (R' - z') \right) - \left( (1 - \tau) c + \lambda \right) \frac{\omega'}{\mu'} + v(\omega', \mu') \right\} d\Phi(z') \]

with \( \omega \equiv b/k, \ v \equiv V/k \)

State of economy: \((\omega, K, \mu, A)\)
FOC for $\omega'$

\[
pg (i) + \frac{\partial p}{\partial \omega'} \left( \omega' g (i) - (1 - \lambda) \frac{\omega}{\mu} \right)
\]

\[
= g (i) E M' \Phi (z^{*'}) \frac{1}{\mu'} \left[ (1 - \tau) c + \lambda + (1 - \lambda) p' \right]
\]
One-period debt, $\lambda = 1$

$$p + \frac{\partial p}{\partial \omega'} \omega' = EM' \Phi (z^*) \left[ ((1 - \tau) c + 1) \frac{1}{\mu'} \right]$$

Proposition:

- Assume $\mu$ i.i.d., $\xi^r = 0$, and no shocks for a long time so that $\mu_{t-1} = \bar{\mu}, \omega_t = \bar{\omega}$
- Then, shock on $\mu_t$ has no effect on $\omega_{t+1} = \bar{\omega}$
Sticky Leverage

Long-term debt, $\lambda < 1$

$$pg\,(i) + \frac{\partial p}{\partial \omega'}\left(\omega'g\,(i) - (1 - \lambda) \frac{\omega}{\mu}\right)$$

$$= g\,(i) EM' \Phi\left(z^*\right) \frac{1}{\mu'} \left[\left((1 - \tau) c + \lambda\right) - p' (1 - \lambda)\right]$$

Proposition

- Assume $\mu$ i.i.d., $\zeta' = 0$, and no shocks for a long time so that $\mu_{t-1} = \bar{\mu}, \omega_t = \bar{\omega}$
- A negative shock on $\mu_t$ increases $\omega_{t+1} > \bar{\omega}$.

Sticky leverage: high $\omega / \mu \Rightarrow$ high $\omega'$
Optimal Investment and Debt Overhang

FOC for $i$

$$1 - p\omega' = EM' \int_{z'}^{Z^*} \left\{ \begin{array}{c} (1 - \tau)(R' - z') \\ - ((1 - \tau)c + \lambda) \frac{\omega'}{\mu'} \\ + \nu(\omega', \mu') \end{array} \right\} d\Phi(z')$$

Proposition:

- Assume $\mu$ i.i.d., $\xi^r = 0$, and no shocks for a long time so that $\mu_{t-1} = \bar{\mu}, \omega_t = \bar{\omega}, R_{t+1} > \bar{R}$
- Then, shock on $\mu_t$ has no effect on $R$ (and $i$) iff $\omega_{t+1} = \bar{\omega}$
- However if $\omega_{t+1} > \bar{\omega}$ then $R_{t+1} > \bar{R}$
Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>$\beta$</td>
<td>Subjective Discount Factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Risk Aversion</td>
<td>1</td>
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<tr>
<td>$\theta$</td>
<td>Elasticity of Labor</td>
<td>0.63</td>
</tr>
<tr>
<td>$\alpha$</td>
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<td>0.36</td>
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<td>$\delta$</td>
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## Calibration

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<td>$\delta$</td>
<td>Depreciation Rate</td>
<td>0.025</td>
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<tr>
<td>$\lambda$</td>
<td>Debt Amortization Rate</td>
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<td>$\tau$</td>
<td>Tax Wedge</td>
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<td>$\eta_1$</td>
<td>Distribution Parameter</td>
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<td>$\zeta$</td>
<td>Default Loss</td>
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<tr>
<td>$\xi^r$</td>
<td>Fraction of Resource Cost</td>
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</tbody>
</table>
Idiosyncratic Shocks

Use general quadratic approximation to p.d.f.:

\[ \phi(z) = \eta_1 + \eta_2 z + \eta_3 z^2 \]

- Symmetry \( \bar{z} = z = 1 \), and \( E(z) = 0 \)
- One free parameter \( \eta_1 \)
Shocks

VAR process for inflation and productivity

\[
\begin{bmatrix}
a_t \\
\mu_t
\end{bmatrix}
= \begin{bmatrix}
\rho_a & \rho_{a,\mu} \\
\rho_{a,\mu} & \rho_{\mu}
\end{bmatrix}
\begin{bmatrix}
a_{t-1} \\
\mu_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon^a_t \\
\varepsilon^\mu_t
\end{bmatrix}
\]

Estimated values

\[
\Gamma = \begin{bmatrix}
0.98 & -0.094 \\
0.012 & 0.85
\end{bmatrix}
\]

\[
\sigma_a = 0.0074, \sigma_\mu = 0.0045, \rho_{\mu a} = -0.19
\]

AR(1) version:

\[
\rho_a = 0.97, \quad \sigma_a = 0.0070
\]

\[
\rho_\mu = 0.85, \quad \sigma_\mu = 0.0040
\]

\[
\rho_{a,\mu} = \rho_{a,\mu} = \rho_{\mu a} = 0
\]
Inflation shock
### Key Moments

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model AR(1)</th>
<th>Model VAR(1)</th>
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<tbody>
<tr>
<td><strong>First Moments</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Investment/Output, $I/Y$</td>
<td>0.21</td>
<td>0.24</td>
<td>0.24</td>
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<tr>
<td>Leverage, $\omega$</td>
<td>0.42</td>
<td>0.42</td>
<td>0.42</td>
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<tr>
<td>Default Rate, $1 - \Phi(z^*)$</td>
<td>0.42%</td>
<td>0.42%</td>
<td>0.42%</td>
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<tr>
<td>Credit Spread</td>
<td>0.39%</td>
<td>0.39%</td>
<td>0.39%</td>
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<tr>
<td><strong>Second Moments</strong></td>
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<tr>
<td>$\sigma_Y$</td>
<td>1.7%</td>
<td>1.6%</td>
<td>1.7%</td>
</tr>
<tr>
<td>$\sigma_I/\sigma_Y$</td>
<td>4.12</td>
<td>4.22</td>
<td>4.48</td>
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<tr>
<td>$\sigma_C/\sigma_Y$</td>
<td>0.54</td>
<td>0.41</td>
<td>0.43</td>
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<tr>
<td>$\sigma_N/\sigma_Y$</td>
<td>1.07</td>
<td>0.49</td>
<td>0.54</td>
</tr>
<tr>
<td>$\sigma_\omega$</td>
<td>1.7%</td>
<td>1.5%</td>
<td>1.7%</td>
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### Variance decomposition, AR(1)

<table>
<thead>
<tr>
<th></th>
<th>Y</th>
<th>Inv</th>
<th>Cons</th>
<th>Hrs</th>
<th>Lev</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Benchmark, (\bar{\omega} = 0.42)</strong></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>TFP shock (a)</td>
<td>0.63</td>
<td>0.37</td>
<td>0.39</td>
<td>0.60</td>
<td>0.10</td>
<td>0.05</td>
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<tr>
<td>Inflation shock (\mu)</td>
<td>0.37</td>
<td>0.63</td>
<td>0.61</td>
<td>0.40</td>
<td>0.90</td>
<td>0.95</td>
</tr>
<tr>
<td><strong>Low Leverage, (\bar{\omega} = 0.32)</strong></td>
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<td></td>
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<tr>
<td>TFP shock (a)</td>
<td>0.84</td>
<td>0.65</td>
<td>0.83</td>
<td>0.89</td>
<td>0.10</td>
<td>0.01</td>
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<tr>
<td>Inflation shock (\mu)</td>
<td>0.16</td>
<td>0.35</td>
<td>0.17</td>
<td>0.11</td>
<td>0.90</td>
<td>0.99</td>
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<tr>
<td><strong>High Leverage, (\bar{\omega} = 0.52)</strong></td>
<td></td>
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<tr>
<td>TFP shock (a)</td>
<td>0.40</td>
<td>0.21</td>
<td>0.29</td>
<td>0.55</td>
<td>0.03</td>
<td>0.03</td>
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<tr>
<td>Inflation shock (\mu)</td>
<td>0.60</td>
<td>0.79</td>
<td>0.71</td>
<td>0.45</td>
<td>0.97</td>
<td>0.97</td>
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Variance decomposition, AR(1)

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<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td><strong>Benchmark, $\lambda = 0.06$</strong></td>
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<tr>
<td>TFP shock $a$</td>
<td>0.63</td>
<td>0.37</td>
<td>0.39</td>
<td>0.60</td>
<td>0.10</td>
<td>0.05</td>
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<tr>
<td>Inflation shock $\mu$</td>
<td>0.37</td>
<td>0.63</td>
<td>0.61</td>
<td>0.40</td>
<td>0.90</td>
<td>0.95</td>
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<tr>
<td><strong>Long maturity, $\lambda = 0.03$</strong></td>
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<tr>
<td>TFP shock $a$</td>
<td>0.45</td>
<td>0.26</td>
<td>0.65</td>
<td>0.80</td>
<td>0.83</td>
<td>0.01</td>
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<tr>
<td>Inflation shock $\mu$</td>
<td>0.55</td>
<td>0.74</td>
<td>0.35</td>
<td>0.20</td>
<td>0.17</td>
<td>0.99</td>
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<tr>
<td><strong>One period debt, $\lambda = 1$</strong></td>
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<tr>
<td>TFP shock $a$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.01</td>
<td>0.02</td>
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<tr>
<td>Inflation shock $\mu$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.99</td>
<td>0.98</td>
</tr>
</tbody>
</table>
Monetary policy rule

Taylor rule with interest rate smoothing

\[
\hat{r}_t^f = \rho_R \hat{r}_{t-1}^f + (1 - \rho_R) \{ \nu_m \hat{\mu}_t + \nu_y \hat{y}_t \} + \zeta_t
\]

Calibration

\[
\hat{r}_t^f = 0.6 \cdot \hat{r}_{t-1}^f + 0.4 \{ 1.5 \cdot \hat{\mu}_t + 0.5 \cdot \hat{y}_t \} + \zeta_t
\]
Monetary policy shock

![Graphs showing the impact of monetary policy shocks on various macroeconomic variables.](image-url)
Productivity shock
Wealth/capital shock
Conclusion

- Model with nominal long-term debt produces strong inflation non-neutrality without sticky prices
- Key mechanisms: sticky leverage and debt overhang
- Taylor rule implies a significant increase in inflation in response to both low productivity and wealth shocks