

Rules and Discretion: An Empirical Assessment

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“Rules are for the obedience of fools and the guidance of wise men”

Harry Day (RAF Group Captain, B1898-D1977)

1. Introduction

In 1993, John Taylor introduced the notion of a “policy rule:” a linear function relating the central bank’s policy rate to its equilibrium level, and to deviations of its goal variables from their desired levels.² The rule that he posited in his original paper took the simple form

$$r = p + .5y + .5(p - 2) + 2 \quad , \quad (1.1)$$

where r is the central bank-controlled short-term interest rate, p is the rate of inflation in the GDP deflator, y is the percent deviation of real GDP from trend ($100*(Y-Y^*)/Y^*$, where Y^* is trend real GDP), and the constant 2 is meant to be the equilibrium real federal funds rate. The inflation goal in the rule is 2 percent, hence the deviation of inflation from two in the third term on the right-hand side.

As simple as it is, this rule serves perfectly well to introduce a number of concepts that are necessarily entailed in considering how to construct and use a policy rule. These include:

- a. What is the intended status of the policy rule? Is it a guideline or benchmark, or does it hold some claim to “optimality”—i.e., it represents (approximately) the best that monetary policy can do to stabilize inflation and output around their targets?
- b. How important are the unobservable quantities in the policy rule? And how do we estimate these? Do they vary over time, or are they assumed to be constant?³ The key unobservables are
 - i. The inflation goal (during the historical sample examined by Taylor, and in the U.S. until an explicit numerical objective was announced in January 2012);
 - ii. The equilibrium real rate of interest, a critical variable in that it contributes to the “normal” policy rate against which actual policy settings are judged in this simple rule. That is,
 - iii. The trend level and rate of growth of output (and/or the natural rate of unemployment). Numerous papers have been written detailing the host of

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² See Taylor (1993).

³ Clarida (1999) raises these issues in an early discussion about policy rules.

issues involved in inferring this quantity, which almost surely varies over time, for a variety of reasons.

- c. What status do the coefficients [0.5, 0.5] have in the canonical rule? They describe reasonably well the evolution of the federal funds rate from 1987 to 1992, using the data vintages and definitions in Taylor's original paper. But would adherence to such a rule have produced desirable economic outcomes, then or now?
- d. Why is the rule written in terms of realizations, rather than forecasts? The actual policy process at most all central banks involves consideration of a forecast as a key input to the policy decision, in recognition of the time lag between policy implementation and effects on the macroeconomy. Would forecast-based rules be superior to realization-based rules, perhaps for this reason?
- e. While not included in the original paper, when interpreted as a behavioral equation fit to actual data, equation (1.1) would include an error term, reflecting the extent to which the equation does not perfectly capture all of the fluctuations in the federal funds rate:

$$r_t = p_t + .5y_t + .5(p_t - 2) + 2 + e_t \quad (1.2)$$

What is the interpretation of that error term? Does it reflect discretion—a judgmental deviation from the rule? Does it reflect the influence of other variables on monetary policy that are not well-captured by realizations of output and inflation? If we include such variables, do we reduce the size of the error term, because we better capture the actual actions of the policy authority in setting interest rates? Is it reasonable to interpret the e_t as monetary policy “shocks,” in the same vein as Christiano, Eichenbaum and Evans (2005) or Romer and Romer (2004)?

In this paper, we provide empirical evidence that may help us to sort through some of the answers to these questions. But in doing so, we will emphasize that the full answers to these questions are fraught with difficulty and thus still the subject of study by many researchers. It is in part for this reason (as well as others) that we feel it would be unwise in our view to strictly bind a central bank to any such rule.

That said, the contributions of Taylor and others working on policy rules have been essential, and the difficulties mentioned above and studied below do not diminish the value of policy rules in both academic and practical policy settings. Indeed, every central bank strives to set policy systematically, and the essence of empirical policy rules is that they attempt to capture the systematic component of monetary policy. Thus as an empirical benchmark, such rules are quite valuable in the policy processes of many central banks, including the Federal Reserve.

Policy rules are also valuable as perhaps the leading means of representing monetary policy in econometric models. Rules are the most common way of endogenizing the monetary policy instrument in current DSGE and other macroeconomic models employed by central banks around the world. Absent such a clear and empirically-verified way of representing monetary policy, we might be back in the world of estimating ever-shifting money demand equations to represent the role of monetary policy in the macroeconomy. Clearly, policy rules have been a great step forward in this regard.

The goal of this paper is to estimate the systematic component of monetary policy, giving careful consideration to the concerns (a)-(e) raised above. Our best estimates of the systematic component

imply a non-systematic component, which we will use as one measure of discretionary policy.⁴ The non-systematic component has also been widely employed to represent monetary policy shocks, which many have used to identify the impact of policy on real variables and inflation. We will discuss these notions in more detail in what follows. If time-variation in the equilibrium unemployment rate, the real rate, potential growth and the inflation goal is non-trivial, then ignoring this time-variation in estimating the systematic component of policy necessarily contaminates the discretion/monetary policy shock estimates with the time varying components of these variables. We believe it is an improvement to take this time variation into account. However, because the extent and nature of time variation is uncertain, our methods could likewise contaminate the estimated monetary policy shocks if we over-estimate the degree of time-variation. On balance, we are comfortable incorporating such variation—with care—but it is important to recognize the econometric risks in doing so.

We focus primarily on forecast-based rules, as distinct from the realization-based rules in Taylor's original work. The reasons for this focus are twofold. First, as a description of the systematic component of monetary policy, forecast-based rules better reflect current central bank practice. Such rules also come closer to the objective-based policy described in Svensson (2003): the policy instrument is set so as to close expected gaps between goal variables and their target values. Second, the use of forecasts as the variables to which the central bank responds allows us to capture the host of information that is reflected in the forecasts, as distinct from the limited information reflected in a few (lagged or current) realized variables. That makes forecast-based rules less susceptible to one criticism that simple rules face: Central banks in practice respond to much more than the realization of two key variables.

To preview our main conclusions, we find that:

- Explicitly forward-looking policy rules based on Federal Reserve staff forecasts appear to capture the systematic component of monetary policy quite well. Such rules dominate backward-looking, realization-based policy rules in explaining the history of federal funds rate actions, from 1966 to 2007;
- Accounting for measurement bias and embedded policy rate assumptions in forecasts, we find substantial and significant responses to inflation, unemployment, and output growth gaps;
- A substantial role for interest rate smoothing remains after accounting for time-variation in unobservables, and after employing forecasts and instrumenting them for reasons articulated below;
- Estimating unobserved quantities—the equilibrium real rate, the natural rate of unemployment, the potential growth rate of output, and (in some periods) the inflation goal—is a nontrivial exercise that can significantly alter conclusions about historical policy responses, as well as whether imperfect knowledge of these unobservables significantly led policymakers astray, as discussed in Orphanides (2003), and Romer and Romer (2002).
- We examine deviations from the systematic component of policy, which might be associated with “discretionary” monetary policy, and with monetary policy “shocks”. We show that

⁴ As we discuss below, discretion could enter the policy process in other ways, but this is a conventional way of delineating between systematic and discretionary policy.

some of these deviations can be explained by variables that might not be fully captured either by forecasts or by lagged realizations of the key variables;

- In addition, we compute via two methods the extent to which such deviations might affect economic outcomes. The first-order observation is that these deviations are (a) serially uncorrelated and (b) relatively small, with a standard deviation of 0.5 ppt or less. In addition, using simple empirical methods, we find that the contribution of these shocks to the variance of unemployment or inflation is modest at best.
- Finally, we develop some evidence that an estimate of “optimal” monetary policy implies policy rate settings that differ relatively little from realized policy. However, to the extent that optimal policy differs from realized policy, the results suggest that, if anything, the Fed has demonstrated a slight bias towards tightening more than the optimal policy would prescribe. This is more the case in the 1980s than in more recent years.

Overall, we emphasize that it is of course desirable for central banks to act in a systematic and (where possible) predictable manner.⁵ Policy rules—especially estimated rules—attempt to capture and quantify that systematic component. But two overarching caveats remain: (1) Estimating the systematic component of policy requires taking a stand on some key underlying variables—the unobservables enumerated above—and doing so is a nontrivial matter, for both theoretical and empirical reasons. As we show below, estimates of these critical inputs to monetary policy are therefore clouded in uncertainty. As a consequence, they also impart uncertainty to the task of estimating historical monetary policy rules and shocks.⁶ To be sure, uncertainty surrounding these inputs complicates target-based, optimal, or discretionary policy as well. However, in recognition of this uncertainty, strict adherence to simple linear rules seems unduly restrictive and risky; (2) No policy rule, estimated or posited, should be taken as the best that a central bank can do, unless it can be shown to be optimal or nearly so by standard criteria. But the only way to test for optimality is in the context of a macroeconomic model, and the uncertainties around the appropriate specification for such a model are considerable. This makes the determination of how well any rule approximates optimal policy subject to considerable uncertainty.

Together, these considerations argue for a “portfolio approach” toward monetary policy, drawing on a variety of benchmarks, model-based prescriptions, and judgment. Overarching this approach might be a “goal-based” policy that uses a variety of tools to choose instrument settings that minimize deviations from the central bank’s goals. A simple version of this kind of policy is discussed in section 5.

2. Estimating key unobservable quantities

As noted above, many of the key quantities involved in describing the systematic component of monetary policy are not directly observable. These include the natural rate of unemployment, the

⁵ An issue we do not take up in this paper is the extent to which the policymakers’ forecasts may be sub-optimal, that is, they inefficiently use information available to them to forecast their goal variables. To the extent that forecast errors are avoidable, this would constitute another source of sub-optimal disruption to the economy that derives from monetary policy. Of course, some of any forecast error is unavoidable. While efficient forecast errors will also disrupt the economy (relative to a world in which policymakers could foresee the future), such errors are of less concern.

⁶ There is a large literature examining time-variation in the systematic component of monetary policy, with little consensus at this point on the nature and extent of time-variation. See, e.g., Boivin (2006), Davig and Doh (2014), Murray, Nikolsko-Rzhevskyy, and Papell (2015), and Sims and Zha (2006).

equilibrium real federal funds rate, the level and growth rate of potential or equilibrium output, and (until recently in the U.S.) the target rate of inflation.

Where available, we use explicit estimates of potential growth and the natural rate of unemployment as published in the staff forecasts that are reported in the Greenbook or, more recently, the Tealbook (TB).⁷ For convenience, we will refer to these forecasts throughout as “TB” forecasts. These are only available starting in 1987 and 1989, respectively, with estimates of the natural rate being reported on a more regular basis from 1993 on. The Fed did not announce its explicit numerical inflation objective until January 2012, so we must estimate the implicit inflation target throughout our sample. For the period that we consider, we also need to infer the TB’s assessment of the level of the real federal funds rate expected to prevail in the longer run.

It is important to note that there exists an inherent trade-off in estimating time-varying values for the desired values of variables that the monetary authority responds to. Simply put, if (say) the inflation target varies more, hewing somewhat more closely to actual or expected inflation, then the implied inflation gap is that much smaller. This in turn implies that, for a given observed movement in the funds rate, one will infer a larger response coefficient to the inflation gap. Conversely, a less variable target will imply larger gaps, and a smaller implied response coefficient. While this does not imply a strict observational equivalence between more (less) variable unobservables and larger (smaller) response coefficients, it does suggest a tradeoff in identifying both time-varying unobservables and policy response coefficients.

We attempt to mitigate this trade-off by putting some plausible structure on the time-variation in the unobservables and, importantly, using additional information that should be useful as a signal that is correlated to the unobservable quantity. In all cases, we use the information in the TB forecasts to infer information about the unobservables. We also use additional variables—implied forward rates on Treasuries, long-term inflation expectations from surveys of professional forecasters, and the federal funds rate itself—to make stronger inferences about the inflation goal and the equilibrium real federal funds rate. In the end, the standard errors around our policy rule estimates are non-trivial, and it is wise to recognize that, in part because of the uncertainty that still attends estimation of time-varying unobservable quantities, recovery of the systematic component of policy actions is a difficult exercise. A corollary to this observation is that, because estimates of these unobservable quantities are required to implement any policy rule, this is another reason to take any estimated rule as guidance, not as law.

Inflation target and natural rate of unemployment

We model both of these unobserved variables as evolving according to a random walk. We rely on the simple intuition that the TB forecasts will embody a tendency for inflation and the unemployment rate to revert toward the inflation goal and the natural rate, respectively. Of course, this error-correction representation for the dynamics of the TB inflation and unemployment rate forecasts is a reduced-form representation of the structural relationships at work in the TB, whereby monetary policy is adjusted (to some extent) so as to ensure that these variables do not deviate too far from their desired values over the forecast horizon, given other shocks to the

⁷ The TB begins reporting the output gap in 1987:Q3. We use the identity that links the change in the output gap to the difference between the growth rate in real GDP and the growth rate in potential GDP to back out the implied estimate of potential growth, quarter by quarter.

economy.

While we use an admittedly simple representation of the TB inflation and unemployment forecast process, we exploit the multiple horizon feature of the TB forecasts to augment the dynamics of inflation and unemployment with transitory factors, which we model as an unobserved stationary component. In other words, we allow the TB forecast for inflation and unemployment to respond to developments that are not captured by lags of inflation or the unemployment rate. For example, the dynamics of inflation in the TB forecast could be affected by the TB projection of real activity and by transitory price-level shocks. Rather than trying to control for each of these factors, we summarize them by means of a stationary state variable. The identification of this component, together with the inflation goal and the natural rate, is made possible by the fact that the TB forecasts are available at multiple horizons.

The basic structure of our exercise is summarized by the equations below. For expositional purposes, we consider three forecast horizons and we label the variable of interest generically by y , with the forecast for period $t+i$ made at time t denoted by $y_{t,t+i}^f$. We denote by y_t^* the TB's assessment of the variable's long-run value (the inflation goal or the natural rate of unemployment) that we are interested in estimating:

$$\begin{aligned}
\Delta y_{t,t+i}^f &= \beta(y_{t,t+i-1}^f - y_t^*) + B(L)\Delta y_{t,t+i-1}^f + \rho^i \eta_t^y, \\
\Delta y_{t,t+i+1}^f &= \beta(y_{t,t+i}^f - y_t^*) + B(L)\Delta y_{t,t+i}^f + \rho^{i+1} \eta_t^y + v_1, \\
\Delta y_{t,t+i+2}^f &= \beta(y_{t,t+i+1}^f - y_t^*) + B(L)\Delta y_{t,t+i+1}^f + \rho^{i+2} \eta_t^y + v_2, \\
y_t^* &= y_{t-1}^* + \varepsilon_t^y, \\
\eta_t^y &= \rho \eta_{t-1}^y + \varepsilon_t^\eta.
\end{aligned} \tag{2.1}$$

Several considerations are in order. The observables in the system are the projected values $y_{t,t+i}^f$ at the different horizons, and the values for y dated earlier than t that may appear as explanatory variables in the autoregressive term $B(L)$. For these values, we use the realizations as reported by the TB at the time the forecast was made, that is, we use real-time values. The unobserved state variables in the system are y_t^* and η_t^y . As already mentioned, the term η_t^y captures stationary factors that affect the projected dynamics of y beyond its lags and y_t^* . We express the dynamics of the forecast in first differences, denoted by Δ , to take into account the unit root in y inherited from y_t^* . The expected change at horizon i , $\Delta y_{t,t+i}^f$, is a function of the deviation of y from y_t^* , with the coefficient $\beta < 0$ capturing the degree of convergence of the projected variable to its long-run value y_t^* . Given the posited random-walk process for this variable, we have that $y_{t,t+i}^* = y_t^*$. In other words, the inflation goal and the natural rate of unemployment are assumed to be constant over the forecast horizon. The description of the evolution of $\Delta y_{t,t+i}^f$ is completed by the inclusion of its own lags and the transitory state variable η_t^y . Given the assumed process for η_t^y , it follows that $\eta_{t,t+i}^y = \rho^i \eta_t^y$. For the next forecast in the horizon, $i+1$, explanatory variables are shifted one period forward, with expectations for the state variables evolving in the way we have just described.

The equation allows for an additional shock ν_1 , which is a way of relaxing the constraints in the assumed dynamics for the temporary state variable η_t^y , given that this variable encompasses economic forces that are likely to play out differently over the forecast horizon according to which driver is most relevant at a certain point in time. The forecast for the next horizon is modeled similarly, with the shock ν_2 allowed to be correlated with ν_1 .

This basic framework is augmented by other measurement variables that may provide additional information about the evolution of y^* . For example, professional forecasters' expectations about long-run inflation will be correlated with the TB assessment of the inflation goal. Thus for inflation, the estimating equations for the time-varying inflation goal are cast in state-space form as

$$\begin{aligned}
\Delta\pi_{t,t+3+i}^f &= \beta_\pi (\pi_{t,t+3+i-1}^f - \pi_t^*) + \mathbf{B}_\pi(L)\Delta\pi_{t,t+3+i-1}^f + \rho_{\pi,1}^{3+i}\eta_t^{\pi,1} + \nu_{t,3+i}, \quad i = 0,1,2, \\
\Pi_t^{10} &= \pi_t^* + \eta_t^{\pi,2}, \\
R_t^4 &= r_t^* + \pi_t^* + \eta_t^{\pi,3}, \\
R_t^9 &= r_t^* + \pi_t^* + c_R + \eta_t^{\pi,3} + \varepsilon_t^R, \\
\pi_t^* &= \pi_{t-1}^* + \varepsilon_t^\pi, \\
\eta_t^{\pi,j} &= \rho_{\pi,j}\eta_{t-1}^{\pi,j} + \varepsilon_t^{\eta,j}, \quad j = 1,2,3.
\end{aligned} \tag{2.2}$$

Inference about the time-varying inflation goal π_t^* uses the TB projected quarterly change in inflation at horizons 3, 4, and 5 whenever available. In keeping with the framework just described in (2.1), we set $\nu_{t,3} = 0$ for all t . The inference also relies on professional forecasters' expected inflation over ten years, Π_t^{10} , with this variable allowed to deviate from π_t^* according to the transitory process $\eta_t^{\pi,2}$, and on the one-year forward Treasury rate beginning four and nine years hence, denoted by R_t^4 and R_t^9 , respectively. The idea behind using forward rates at 4 and 9 year horizons is that movements in these rates should be less affected by business cycle developments and reflect, among other things, changing perceptions about the Federal Reserve's inflation goal. The measurement equation for these variables features an estimate of the real rate of interest expected to prevail in the long run, r_t^* , which we describe later. As with the 10-year inflation expectations, we allow for persistent deviations from the variables' long run values. The constant c_R in the equation for the forward rate expected to prevail 9 years hence takes into account the presence of a risk premium vis-à-vis the forward rate expected to prevail 4 years hence. It is important to note that some of the observables – notably the TB forecast of the change in inflation 5 quarters out, Π_t^{10} , and R_t^9 – are available only for part of the sample that we consider.

The use of information from surveys and market expectations to infer the FOMC's inflation goal is justified by the fact that inflation in the long run is under the control of the central bank. The same does not hold for other variables, whose long-run value is not affected by monetary policy. For this reason, when inferring the natural rate of unemployment we rely only on the TB forecasts as

modeled in (2.1). Specifically, the natural rate of unemployment is the attractor in a system of error-correction equations for unemployment forecasts at multiple horizons:

$$\begin{aligned}
\Delta u_{t,t+3+i}^f &= \beta_u (u_{t,t+3+i-1}^f - u_t^*) + B_u(L) \Delta u_{t,t+3+i-1}^f + \rho_u^{3+i} \eta_t^u + v_{t+3+i}, \quad i = 0, 1, 2, 3 \\
u_t^* &= u_{t-1}^* + \varepsilon_t^u, \\
\eta_t^u &= \rho_u \eta_{t-1}^u + \varepsilon_t^\eta.
\end{aligned} \tag{2.3}$$

The system shows that we are using the TB forecasts for the change in the unemployment rate 3, 4, 5, and 6 quarters out. Again, we set $v_{t,3} = 0$ for all t , and we note that some of the forecasts at the longer horizons are not always available.

Potential GDP growth and the equilibrium real federal funds rate

The reduced-form approach that we use to retrieve estimates of the TB assessment of the FOMC's inflation goal and natural rate of unemployment is justified by the fact that it is hard to infer these variables from more structural relationships. For example, one could think about retrieving the natural rate of unemployment from a Phillips curve relationship estimated on TB forecasts of inflation and unemployment. The tradeoff between inflation and unemployment, however, has not been stable over time and some key features of the specification – notably how inflation expectations are formed – have also changed. Given the many moving parts in this type of relationship, we think that there is an advantage to resort to a more general reduced-form representation, even though stability issues need to be addressed in this context, too. For other key unobservable variables, however, this type of concern is less compelling. For example, the relationship between GDP growth and the change in the TB forecasts has been fairly tight and stable over the sample period that we consider. As a result, this Okun's Law-type of relationship in first differences is exploited to estimate the TB assessment of potential GDP growth. It is also the case that the TB real activity forecast will be responsive to the stance of monetary policy. We thus retrieve an estimate of the equilibrium real federal funds rate by means of an IS-type relationship, albeit still reduced-form, between GDP growth and the real federal funds rate. We continue to assume, as with the inflation goal and the natural rate of unemployment, that these unobservables evolve as a random walk.

The state-space representation used to retrieve potential GDP growth is

$$\begin{aligned}
\Delta u_{t,t+i+j}^f &= \beta_y (\Delta y_{t,t+i+j}^f - \Delta y_t^*) + B_y(L) \Delta u_{t,t+i+j-1}^f + v_{t+i+j}, \quad i = 3, \quad j = 0, 1, 2, 3 \\
\Delta y_t^* &= \Delta y_{t-1}^* + \varepsilon_t^y, \\
\Delta \bar{y}_{t,t+i+j}^f &= \Delta y_t^* + \delta_j (i_{t-1} - \pi_{t-1}^4 - \bar{r}) + \rho_y^{i+j} \eta_t^y + v_{t+i+j}, \quad i = 2, \quad j = 0, 1, 2 \\
\eta_t^y &= \rho_y \eta_{t-1}^y + \varepsilon_t^\eta.
\end{aligned} \tag{2.4}$$

The first relationship in the system illustrates the use of Okun's Law relationships, “stacked” at multiple forecast horizons, linking the change in unemployment with the deviation of output growth from potential growth. Potential GDP growth is assumed to evolve as a random walk. While it can be shown that in practice our inference of the TB assessment of potential mostly comes from these relationships, we also append reduced-form IS equations that relate the deviation of projected GDP

growth from potential to the realized real federal funds rate, where i is the nominal value of the federal funds rate and π^4 is a 4-quarter moving average of the real-time inflation prevailing in the quarter before the forecast was made. In these equations, the TB forecast of GDP growth is taken as a 2-quarter average, so that $\Delta\bar{y}_{t,t+3}^f = 0.5*(\Delta y_{t,t+2}^f + \Delta y_{t,t+3}^f)$. This makes identification of the interest elasticity parameter δ_j more straightforward. Since we rely on realizations rather than forecasts of the real federal funds rate, in the IS equations the responsiveness of real activity to interest rates will differ at different forecast horizons. Needless to say, developments other than the stance of monetary policy will also affect the TB's GDP growth forecast. For this reason, we include as previously a transitory state variable η^y to capture these factors. In keeping with the same framework as before, we also have that $v_{t,3} = v_{t,2} = 0$ for all t .

In the state-space system (2.4), the equilibrium real federal funds rate is assumed constant at \bar{r} . This assumption does not appear to be crucial when inferring the TB assessment of potential because, as we have already mentioned, it is the Okun's Law relationship that appears to provide most of the signal. In order to estimate a time-varying equilibrium real federal funds rate, we use the estimates of the inflation goal, the natural rate of unemployment, and potential GDP growth from the state-space systems that we have already described. These are fed into an estimated monetary policy rule to infer the TB assessment of the equilibrium real federal funds rate. To improve identification, we use as in (2.4) IS curves linking the projected deviation of GDP growth from potential to the gap between the real federal funds rate and its equilibrium value, where the equilibrium real rate is assumed to follow a random walk. Critically, we assume that the level of the federal funds rate likely provides information about the equilibrium real rate of interest, in the context of a policy rule in which the funds rate moves with some inertia and responds to TB forecasts. Thus the state-space system takes the following form

$$\begin{aligned}
\Delta\bar{y}_{t,t+i+j}^f &= \Delta y_t^* + \delta_j(\bar{f}_{t-1} - \pi_{t-1}^4 - r_t^*) + \rho_y^{i+j}\eta_t^y + v_{t+i+j}, \quad i = 2, \quad j = 0,1 \\
\eta_t^r &= \rho_r\eta_{t-1}^r + \varepsilon_t^\eta, \\
r_t^* &= r_{t-1}^* + \varepsilon_t^r, \\
\bar{f}_t &= \rho_1\bar{f}_{t-1} + \rho_2\bar{f}_{t-2} \\
&+ (1 - \rho_1 - \rho_2)[r_t^* + \pi_t^* + \alpha_\pi(\pi_{t,t+4}^{4,f} - \pi_t^*) + \alpha_u(u_{t,t+4}^f - u_t^*) + \alpha_{dY}(\Delta y_{t,t+4}^{4,f} - \Delta y_t^*)] + \varepsilon_t^{MP},
\end{aligned} \tag{2.5}$$

where $\pi_{t,t+4}^{4,f}$ and $\Delta y_{t,t+4}^{4,f}$ denote the TB projections of average inflation and GDP growth over the first 4 quarters of the forecast horizon, respectively. In the policy rule, $u_{t,t+4}^f$ is the TB forecast of the unemployment rate expected to prevail 4 quarters out.

The systems (2.2)-(2.5) provide us with the elements that we need to estimate a monetary policy reaction function. In what follows, we first illustrate the data and highlight some important issues with estimation. We then discuss our estimation results and what the results imply in terms of rules versus discretion in monetary policymaking.

Data and Estimation

We consider TB forecasts at quarterly frequency. We take the earliest TB forecast in the quarter. In the 1970s, when staff forecasts were more frequent we follow the same approach but for cases in which the first forecast in the quarter is not populated enough. In those instances, we take the second forecast in the quarter, which during that period often covers a longer horizon. As already mentioned, whenever actual data for inflation, the unemployment rate, and output growth are used, we take the values as reported in real-time in the TB. In forecasts and actual data, inflation is measured as the quarterly percent change at an annual rate of the GDP deflator through the end of 1985. From 1986 to 2005, the inflation measure is based on the core CPI. After that, inflation is measured by the core PCE deflator. Output growth is given by the quarterly percent change at an annual rate of real GNP until 1991, and of real GDP thereafter. For the federal funds rate, we use the average effective federal funds rate prevailing in the week after the meeting the TB forecast was prepared for. Estimated forward Treasury rates are from Gurkaynak, Sack and Wright (2006).⁸ Long-run inflation expectations are a combination of the Blue Chip and Livingston surveys over the period 1979:Q4 to 1991:Q4.⁹ We interpolate the data whenever observations are missing. From 1992 to 2005, we use the median value from the Federal Reserve Bank of Philadelphia's Survey of Professional Forecasters. After 2005, we use the FRB/US series for long-run inflation expectations, which translates the Survey of Professional Forecaster's measure, based on the CPI, into a PCE deflator equivalent.

Important issues arise in the estimation of the state-space systems (2.2)-(2.5). Stability over the sample period we consider is an important concern, and for this reason estimation is sometimes split between a period that covers the late 1960s and the 1970s, and a more recent period going from the 1980s to 2007. We stop the estimation sample in 2007 and thus exclude the Great Recession and ensuing recovery because over a large portion of that period the policy rate was constrained at the zero-lower-bound. When estimating the policy reaction function, we exclude the period 1979:Q4-1982:Q3, as this "nonborrowed reserves operating procedure" period explicitly allowed for much greater volatility in the federal funds rate, and might not be properly construed as a period in which the short-term interest rate is the operating instrument.

We model the unobservables – the inflation goal, the natural rate of unemployment, potential GDP growth and the equilibrium real federal funds rate – as random walks. We apply the Kalman filter separately to each of the systems (2.2)-(2.5) to obtain estimates of these unobservables. The variance of the error term associated with the random walk processes is especially important and it is not always pinned down very precisely in our estimation. We resort to different ways of addressing this issue. The TB has been providing its assessment of potential GDP growth and of the natural rate of unemployment since the late 1980s. For these two variables, we calibrate the variance of the random walk process so as to generate a path for potential GDP growth and the natural rate of unemployment that is consistent with the evolution of the published TB assessment for these two variables.

Figures 1 and 2 provide a depiction of our inference of the TB assessment of the natural rate of unemployment and of potential GDP growth, respectively. These estimates, with the associated two standard errors bands, are plotted against the published TB values when available in the latter part of the sample. Also plotted in the figures are the most recent CBO estimates of the natural rate

⁸ These data are available at <http://www.federalreserve.gov/pubs/feds/2006> and are updated periodically.

⁹ The data are maintained by the Federal Reserve Bank of Philadelphia. See <https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/historical-data/inflation-forecasts>.

of unemployment and potential GDP growth for the period in question. As concerns the natural rate of unemployment, we estimate the state-space system (2.2) over the entire sample, 1966:Q4 to 2007:Q4.¹⁰ The results of this exercise are depicted in Figure 1. There is considerable uncertainty around our estimates. Reported TB estimates of the natural rate of unemployment are relatively few until the early 1990s. The match between our and the reported TB estimates is relatively close, with deviations that are persistent but do not indicate, overall, a bias. Our estimates show considerably more volatility than the current CBO vintage. They are below the CBO estimate at the beginning and at the end of the sample.¹¹ By the end of the 1970s, our estimates start to be persistently above the CBO's, with the difference especially pronounced in the early 1980s.

Figure 2 reports the same exercise for potential GDP growth. Preliminary estimates suggest that the relationship between TB forecasts of GDP growth and the unemployment rate has been relatively stable over time. For this reason, we estimate the state-space system (2.3) over the whole sample. The correspondence between our estimates and the published TB estimates starting in 1987 is fairly close, with no evident signs of bias. The current CBO estimate of potential GDP growth is widely different, but unlike the unemployment rate, GDP data can, and have been, revised significantly.

As concerns the inflation goal, stabilization of private forecasters' long-run inflation expectations around 2 percent since the late 1990s is consistent with the presence of an implicit but well understood and credible inflation goal. Inference of the inflation goal is conducted separately over two subsamples, 1966:Q4 to 1985:Q4, and 1986:Q1 to 2007:Q4.¹² For the more recent subsample, the variance of the process for π^* is left unconstrained. For the earlier subsample, we calibrate a value for the variance of the innovation in π^* that generates estimates of the inflation goal which, by the end of 1985, are similar to what is being estimated at the beginning of the next subsample. The calibrated value for the variance generates an estimate of the inflation goal that, over the period 1966 to 1985, is less variable than what we would have estimated had we left the parameter unconstrained. Our inference of the TB assessment of the inflation goal is plotted in Figure 3. Perhaps not too surprisingly, the estimate increases over the course of the mid- to late-1970s, before slowly reverting back to 2 percent. The difference in the uncertainty surrounding the estimates in the first and the second subsample is especially striking. In essence, for the period from 1986 to 2007 the state-space representation (2.2) provides an estimate of the TB assessment of the inflation goal that is essentially the same as private forecasters' long run expectations.

Given the estimated values for potential GDP growth, the natural rate of unemployment, and the inflation goal, it is then possible to estimate the state-space system (2.5). The system provides an estimate of the monetary policy reaction function, and given that this is the main focus

¹⁰ The first few observations are used as initial values in the estimation, and as a result in Figure 1 we report estimates of

¹¹ At the beginning of the sample, the TB forecasts typically cover a very short horizon. As a result, it is possible that those forecasts did not show meaningful reversion to the equilibrium unemployment rate. For this reason, the estimated assessment of the natural rate of unemployment in the late 1960s and early 1970s is likely even more uncertain than the standard error bands would suggest.

¹² The state-space representation (2.2) also includes the equilibrium real federal funds rate as an unobservable variable. We first estimate (2.2) with a constant equilibrium real federal funds rate. The estimates that we back out for the inflation goal are then used together with the estimates for potential GDP growth and the equilibrium unemployment rate to obtain a time-varying estimate of the equilibrium real federal funds rate in (2.5). This estimate is then used to re-estimate the system (2.2).

of our analysis, we defer discussion of the reaction function’s estimated parameters to the next section. Here, we just comment on the equilibrium real federal funds rate that we back out from the estimation of (2.5). We choose a value for the variance of the innovation in r^* that we view as conservative. Specifically, for the period covering 1981:Q4 to 2007:Q4, the calibrated variance is about half of the variance of the change in r^* as estimated according to the Laubach and Williams (2003) procedure over the same period. This provides a dynamics for the equilibrium real federal funds rate that is relatively slow-moving over time. The calibrated value for the variance of the innovation in r^* for the earlier subsample, 1966:Q4 to 1981:Q3 is larger but in line with the Laubach and Williams estimates.

The estimated TB assessment of the equilibrium real federal funds rate is plotted in Figure 4. We here report estimates that cover the entire sample period, but in the next section when we discuss the estimated parameters in the policy reaction function, we will exclude the nonborrowed reserves operating procedures period. The figure shows that the estimated TB assessment of the equilibrium real federal funds rate declines over the course of the 1970s and then rises sharply in the early 1980s. Since then, the estimate has first declined and then, from the mid-1990s on, stabilized to a value between 2 and 2½ percent. The behavior of the estimate in the earlier part of the sample can be shown to be in sharp contrast with the estimates in Laubach and Williams, which would call for a higher equilibrium real rate over the 1970s.¹³ It should also be noted, however, that low estimates of the equilibrium real federal funds rate over the 1970s apply when estimating versions of Laubach and Williams which impose less structure on the dynamics of the equilibrium real federal funds rate.

We exclude the period 1979:Q4-1982:Q3, as this “nonborrowed reserves operating procedure” period explicitly allowed for much greater volatility in the federal funds rate, and might not be properly construed as a period in which the short-term interest rate is the operating instrument. In addition, we observe some differences in the policy rule coefficients from 1968-1979 and 1983-2008, so we estimate these periods separately.

3. Estimating policy rules

We will focus primarily on the period since 1982:Q3, although we will also present results for the pre-1980 period. Estimating the system (2.5) from 1983:Q1 to 2007:Q4 via maximum likelihood, taking as given the estimates of $[\pi_t^*, u_t^*, \Delta y_t^*]$ derived from systems (2.2), (2.3), and (2.4), we obtain the following estimates of the key policy rule parameters:¹⁴

The coefficients are both sizable and significant. The degree of interest rate smoothing is noticeably smaller in the earlier sample (the sum of the lag coefficients is 0.48 versus 0.89 in the later sample). In both cases, we estimate sizable responses to inflation and unemployment. In preliminary estimates, we fail to develop a sizable or significant coefficient on real GDP growth in the 1966-79 period, so it is omitted from the specification going forward. As a first pass, these policy rules appear sensible. They suggest moderate and stabilizing policy responses, taking into account the time-variation in key unobservables. We address a number of potential concerns and additions to the rules in the following subsections.

¹³ On this point, see also Orphanides and Williams (2002).

¹⁴ The period 1982:4 appears to be an influential observation. Including it makes a significant difference to most of the parameter estimates, for reasons that are not entirely clear. We omit it from the sample in the remainder of the paper.

Policy rules and the policy assumption embedded in the forecasts

If the forecasts employed in estimating the policy rule incorporate a funds rate path that closes the gaps between policy variables and their targets, and if policymakers plan to take actions that are consistent with that funds rate path, then their actual actions will appear to *over*-respond to the gaps implicit in the forecasts. This is a problem with estimating policy rules from forecast data, as distinct from rules that rely on actual lagged data, or on instrumented future values as proxies for forecasts.¹⁵

In addition, while Federal Reserve staff forecasts are routinely examined as a part of the policy process in the United States, there is no guarantee that voting members of the FOMC strictly adhere to these forecasts in determining how to vote for the policy action. Thus the TB forecasts may properly be viewed as measured with error, relative to the forecasts that individual FOMC members have in mind at the time of their vote.

For both of these reasons, instrumenting the forecasts using information that does not contain the assumed forward path of interest rates should help to mitigate both potential issues with the forecasts in the model above. Note that these two effects imply biases in the opposite direction—a bias upward due to the embedded policy path, and a bias downward due to measurement error. The sign of the net bias is ambiguous. For these reasons, we consider rules that instrument for the forecasts of the key variables.

To implement this strategy, we employ two methods. First, we compute fitted values for the forecasts using OLS regressions of the forecasts on candidate instruments. The instruments for this subsection are four lags each of the real-time estimates of inflation, the unemployment rate, and real GDP growth. Instrument relevance is quite good: the first-stage F-stats are 74.2 for inflation (p -value 0.000), 115.6 for unemployment (p -value 0.000), and 8.61 for real GDP growth (p -value 0.000).

We then substitute these fitted values into the system used to produce the results in Table 1, correcting the standard errors in a second step as described in the footnote.¹⁶ As a check on this method, we run GMM on the same policy rule, using the same instrument set, but taking all of the unobservables as fixed regressors. The results for both methods are presented in Table 2.

The estimates are not identical to the OLS estimates, but they are qualitatively similar. Interest rate smoothing is significant, with the sum of the $\rho_i = 0.59$ and 0.85 respectively. Response coefficients are sizable and precisely estimated, although the GMM estimates for the unemployment and output growth responses are somewhat larger than those from the state-space estimation. This upward revision might suggest that measurement error (which would normally bias these

¹⁵ Clarida, Gali, and Gertler (1999 and 2000) estimate a forward-looking monetary policy rule for the U.S. by instrumenting the expected realizations of inflation and the output gap.

¹⁶ The standard errors are corrected by recalculating the sum of squared residuals using the estimated coefficients and the actual observations for inflation, unemployment and real growth. This sum of squared residuals is then used to rescale the variance-covariance matrix of the policy rule parameter estimates, and the correct standard errors are taken as the square root of the diagonal of this rescaled variance-covariance matrix. The same procedure is followed to correct the standard errors in Table 3 for method 1.

coefficients downward) is more important than the incorporation of an active funds rate path in the forecast.

Interestingly, the estimates for both samples imply a response to inflation that is greater than one. A fairly common narrative suggests that failure to adhere to the “Taylor principle”—a funds rate response that more than matches the increase in inflation—was responsible for the rise of inflation in the 1970s. These estimates, like the ones in Orphanides (2004), do not support that view.

On interest-rate smoothing

The persistent presence of what appears to be interest rate smoothing—a sizable and precisely-estimated coefficient on the lagged federal funds rate—suggests that some often-mentioned non-smoothing explanations for the presence of the lagged interest rate are not supported here. First, one rationale for the lagged interest rate in policy rules that are estimated on realized, rather than forecasted data, is that it proxies for the response of the funds rate to many lags of inflation and output.¹⁷ This might especially be the case if one could proxy well for forecasts of inflation and output by using many lags of these variables. But the use of forecasts, which should incorporate whatever information is available in the lags of inflation and output, makes this explanation for the presence of the lagged interest rate less compelling.

In addition, one might think that the lagged interest rate is instead proxying for the low-frequency movements in the funds rate that are due to time-variation in its equilibrium level. But here we have allowed substantial time-variation in the equilibrium real rate and the inflation goal (the two components of the equilibrium nominal funds rate), so again this explanation of interest rate smoothing is less compelling in this context.

Alternatively, the lagged federal funds rate might stand in for important time-variation in the response coefficients in the policy rule. Our estimates suggest some difference between the responses in the 1970s and the later sample. The system in (2.5) can incorporate such a feature, adding state variables for the time-varying responses of the funds rate to the goal variables. Results from this exercise (not shown) indicate a very modest degree of time-variation in the responses, with quite wide standard errors, and average estimates that are essentially the same as those presented in the tables above.¹⁸

Finally, it has been argued that the lagged federal funds rate proxies for a non-systematic but persistent component of, as discussed in Rudebusch (2002). The possibility of a persistent “shock” to the monetary policy reaction function can be easily accommodated in our system (2.5). Results from this exercise (not shown) are not supportive of this view, with estimates for both subsamples

¹⁷ One can see this simply by considering the simple equation $ff_t = \rho ff_{t-1} + \beta x_t + e_t$, a much-simplified policy rule. This equation can be equivalently written as $ff_t(1 - \rho L) = \beta x_t + e_t$, where L is the lag operator. Inverting the lag polynomial on the left-hand side and expanding yields an equation linking the federal funds rate to a long moving average of x_t : $ff_t = \beta[x_t + \rho x_{t-1} + \rho^2 x_{t-2} + \dots] + e_t$, where the weights decline geometrically at the rate ρ .

¹⁸ For example, the response for inflation varies between 3 and 4, with standard error bands that range from -2 to 8. In part due to the “pile-up” problem, we constrain the variance of one of the response parameters, estimating the others freely. After searching over a wide range of constrained variances, we find that the results are not very sensitive to the value chosen.

indicating a degree of persistence in the non-systematic component that insignificantly different from zero, both in statistical and in economic terms.

The J -statistic of the overidentifying orthogonality restrictions fails to reject at any conventional level of significance, suggesting that the instruments are valid according to this criterion. As shown above, the instruments easily pass simple relevance tests as well. The next section examines the role of lagged real-time data in explaining the forecasts, which will indirectly confirm the validity of these instruments for estimating the policy rule coefficients.

The results for the 1966-1979 period reveal that interest rate smoothing is much less prominent a feature during this period. The response to inflation is smaller, but significantly greater than one, and the response to the unemployment gap is about the same size as the later period. The response to GDP growth is estimated to be insignificantly different from zero, and is thus excluded from the specification. Apart from less interest rate smoothing, it is not clear that the earlier period represents a time of vastly different monetary policy responses, in contrast to some narrative about monetary policy during the period.

The role of lagged (real-time) information versus forecasts

As shown in equation (1.1), the original Taylor (1993) policy rule is written in terms of observed variables, rather than forecasts. While the rule estimated above performs well, in the sense of obtaining precisely estimated coefficients and explaining the systematic component of federal funds rate decisions by the FOMC, it is possible that (real-time) lagged observations could improve or dominate the forecasts in this rule. This section addresses that possibility.

We re-estimate the policy rule system from 1983-2007 including real-time estimates of lagged inflation, unemployment, and GDP growth, along with the instrumented Tealbook forecasts for the same variables. These lags are the most recent observations that are considered (preliminary) data by the Fed staff—that is, they are not forecasting for these periods. Preliminary estimates show that the improvement in the likelihood from including four lags each of the TB staff estimates of lagged inflation, unemployment and output growth is insignificant (excluding all eleven variables yields a likelihood ratio test with p -value of 0.168). A single lag of real GDP growth appears to be quite significant, as shown in the table.¹⁹

Table 3 shows the policy rule estimates for the system that includes the single real-time lagged value of GDP growth. The estimated coefficients on the other variables in the policy rule are a bit different from those estimated without the lagged information. While the inflation coefficient is unchanged, the unemployment coefficient, while sizable, is smaller than previously estimated. The coefficient on expected four-quarter growth is now small and insignificant, while that on lagged growth is larger and significant. Still, the overall message from the data is that the forecasts appear to dominate the lagged information in explaining the federal funds rate over this sample period.

¹⁹ This result is confirmed by running a simple omitted variables test on the residuals from the system estimates of the policy rule equation. That is, the residuals are regressed on the lagged real-time variables, along with the TB forecasts that enter the original system. Jointly, the lagged variables are insignificant. The one coefficient that enters significantly, after paring down the specification, is the lagged GDP growth variable, and its estimated coefficient is nearly the same as that estimated with the system approach. The adjustments to the included TB forecast variables are small and mostly insignificant.

This result also allows us to answer the question as to what are valid instruments for the forecasts in the policy rule. The results in Table 3 suggest that lagged real-time data appear (for the most part) not to directly influence the federal funds rate, once forecasts are taken into account, and thus may be valid instruments for the forecasts. The preceding section tested the orthogonality restrictions for instrumented versions of the policy rule, and failed to reject.

The results for the relative importance of lagged data versus forecast differ for the 1966-1979 period. While it is more difficult to draw firm conclusions given the limited number of observations in this period, it appears that the Fed responded more significantly to lagged data in this period than in the sample from 1983 forward. The results in Table 3a, which uses lagged real-time data as instruments, suggest that lagged inflation and unemployment play a bit more of a role in this period than in the later period.²⁰ The sum of the coefficients on forecast and lagged inflation and unemployment are about the same as those estimated in Table 2 above, but some of the response comes from the lagged realizations of inflation and unemployment. Still, the results suggest that the larger input to policy decisions resides in the forecasts.

Principal components of large dataset as proxies for omitted information

Some of what we might impute to “discretion” may reflect systematic responses of the funds rate to data not well captured by the TB forecasts or lagged realizations of high-level macroeconomic variables. To examine this possibility, we construct sets of principal components from a large set of over 200 macroeconomic and financial market variables. The description of the data appears in the data appendix; they include real variables (GDP and components, industrial production, labor market variables), wage and price variables (several price indexes and deflators, several wage measures, disaggregated CPI and PPI data), and financial variables (interest rates, risk premia, equity prices, exchange rates, flow of funds consumer and business credit data). These data are included to measure influences on the funds rate not well-captured by the modal staff forecast, such as measures that might presage financial instability, measures that capture uncertainty around the forecast, volatility in financial markets or the possibility of tail events, and so on.

For this exercise, we take the unobserved variables (potential growth, NAIRU, equilibrium real rate, inflation goal) as given, and estimate the policy rule using lagged actual data (from the TB dataset) and lagged principal components as instruments. To the extent that information in the principal components are reflected in the forecasts, using them as instruments will improve the identification of the rule. If in addition, the principal components enter as independent regressors, this suggests that some of the unexplained component in the policy rules estimated above in fact reflects the systematic response of the funds rate to other variables that are relevant to the policy decision.

The final specification, arrived at after some pre-testing of specifications with more principal components as regressors, is presented below. Note that in this specification, the influence of lagged real GDP growth goes to zero, with a p -value of 0.65, so it is excluded.

²⁰ System-based estimates, not shown, confirm the results from the GMM estimates in Table 3a.

The fact that not all principal components enter as regressors implies that some may be valid instruments for the forecasts, a notion that is validated by the small value of the J -statistic in the bottom panel of the table. In these results, the inclusion of these principal components as instruments improves the precision of the estimated policy responses, and also yields responses that are more plausible in magnitude. The standard deviation of the residuals—the “discretionary” component of policy—is further reduced, although not dramatically (0.44 vs 0.49).

Empirical importance of time-variation in the unobservables

It would be difficult to claim on *a priori* grounds that the equilibrium real rate of interest, the natural rate of unemployment, or the potential growth rate of the economy have not changed over the past 50 years. One could debate more vigorously the constancy of the Fed’s inflation goal, but it seems likely that has changed as well. Still, it may be of interest to compare the policy rule system with a system in which there is no time-variation in these quantities.

To do so, we estimate the system (2.5) without time-variation in the unobservables, which is equivalent to setting the variances of the corresponding random walk processes for $[\pi^*, r^*, u^*, \Delta y^*]$ to zero. The estimated policy rule, employing instruments as described above, is

$$ff_t = 1.11ff_{t-1} - 0.23ff_{t-2} + (1 - 1.11 + 0.23)[3.3 + 3.9\pi_{t,t+4}^{4,f} - 2.3u_{t,t+4}^f + 3.9\Delta y_{t,t+4}^{4,f} - 7.9]$$

The log-likelihood declines by more than eight log points, yielding a likelihood ratio statistic (distributed χ^2 with four degrees of freedom) with a p -value of 0.027, rejecting these constraints.²¹

4. Quantifying “discretion”: How big is it? How much does it matter?

Estimates of monetary policy shocks from the policy rules

The preceding estimates of the systematic component of monetary policy may be of interest in their own right, but they also provide estimates of the “non-systematic” component of policy, which one might interpret as discretionary monetary policy. How large are these estimated discretionary components, and how might we assess their impact on the economy?

Figure 5 presents the fit of the policy rule in Table 4, along with the residuals—an estimate of the discretionary component of monetary policy, widely referred to in the literature as monetary policy shocks.²²

The figure suggests informally that one can take these shocks to the policy rule as *iid*, as there are few episodes of obvious serial correlation in their history. There is a short string of same-signed shocks in the early 1990s, during the recovery from the 1990-91 recession. It is also of

²¹ The response coefficients are implausibly large, and significantly larger than those estimated in tables 2-3, which might suggest that the imposition of constancy on these unobservables exacerbates the measurement error problems discussed above.

²² Note that some of the so-called “systematic” component of monetary policy, as estimated by the average responses in the policy rule, could also be considered discretionary, to the extent that it deviates from sensible or (perhaps) optimal policy. We explore the difference between estimated systematic policy and optimal policy in the next section.

interest that the fit of the model during the onset of the 2001 recession is quite good—even though this is a period of relatively rapid cuts in the funds rate, when one might suspect that estimated interest rate smoothing would cause the rule to lag behind the actual. But this is not the case, due to relatively large estimated responses to the deteriorating forecast for unemployment.

As indicated in Figure 6, there is little statistical evidence of autocorrelation in the estimated shocks to the policy rule. The autocorrelations (with the exception of the fifth lag) lie well within the two-standard error band, and the Ljung-Box Q(12) statistic for the first twelve sample autocorrelations develops a p -value of 0.15.

Overall, departures from the systematic component of the estimated rule are relatively small, and decreasing in variance over this latter sample. As for the earlier sample, the same is true, although the average size of shocks is larger, as shown in the Figure 7. Ocular econometrics might suggest evidence of autocorrelation, especially in the period from 1976 to 1979. But the sample autocorrelations for this series are presented in the Figure 8 and, as the figure indicates, there is no evidence of significant autocorrelation in these estimated shocks to the policy rule. The Ljung-Box Q(12) statistic here develops a p -value of 0.18.

Contribution of policy shocks to unemployment and inflation variance

How much of the variance of unemployment or inflation could these shocks plausibly account for? We consider a simple structural VAR model that includes the unemployment gap, the inflation rate, and the federal funds rate, along with identified monetary policy shocks from the exercises above. The model directly estimates the effects of current and lagged monetary policy shocks on the unemployment gap and the inflation rate, along with the reduced-form effects of lags of all three variables on the same. It imposes a unit coefficient on the impact of the current monetary policy shock on the federal funds rate. Each of the three VAR equations also includes a reduced-form shock $e_t^k, k = [\tilde{u}, \pi, ff]$. Thus the model takes the following form:

$$\begin{aligned}\tilde{u}_t &= b_{U0}^{MP} MP_t^{shk} + b_{U1}^{MP} MP_{t-1}^{shk} + b_{U2}^{MP} MP_{t-2}^{shk} + \sum_{i=1}^3 \alpha_{Ui} \tilde{u}_{t-i} + \sum_{i=1}^3 \beta_{Ui} \pi_{t-i} + \sum_{i=1}^3 \gamma_{Ui} ff_{t-i} + e_t^U \\ \pi_t &= b_{\pi 0}^{MP} MP_t^{shk} + b_{\pi 1}^{MP} MP_{t-1}^{shk} + b_{\pi 2}^{MP} MP_{t-2}^{shk} + \sum_{i=1}^3 \alpha_{\pi i} \tilde{u}_{t-i} + \sum_{i=1}^3 \beta_{\pi i} \pi_{t-i} + \sum_{i=1}^3 \gamma_{\pi i} ff_{t-i} + e_t^\pi \\ ff_t &= MP_t^{shk} + \sum_{i=1}^3 \alpha_i \tilde{u}_{t-i} + \sum_{i=1}^3 \beta_i \pi_{t-i} + \sum_{i=1}^3 \gamma_i \Delta Y_{t-i} + e_t^{ff}\end{aligned}\quad (4.1)$$

We include lags of the policy shocks in the dynamics of unemployment rate and inflation. These equations also control for lags of the federal funds rate, which is given by the sum of the systematic component of policy and the policy shock itself. As a result, the presence of lags of the monetary policy shocks allows the shocks to affect unemployment and inflation in a way that can potentially differ from the impact of the systematic component.

We estimate the coefficients for this model via OLS (the MP shocks should be uncorrelated with the other regressors by construction), taking the MP shocks as fixed regressors.²³ The

²³ While this is a constructed regressor, we are less concerned with the computation of standard errors here than in approximating the transmission of MP shocks to unemployment and inflation.

unemployment rate gap takes the CBO estimate as the measure of the natural rate of unemployment. The inflation measure is the quarterly percent change in the core PCE deflator. We illustrate here results for the period 1983:Q1 to 2007:Q4. We compute the impulse response to a one-time realization of the identified monetary policy shock of one-standard-deviation size, tracing its impact on the system. The variance contribution is then computed as the cumulative sum of squared impulse responses. Finally, we compute the contribution to the variance of each variable by expressing the MP shock contribution relative to the k -step-ahead variance of each variable, computed from this model according to a procedure outlined in the second appendix. Figure 9 displays the variance contributions computed in this manner, with 90 percent confidence bands.²⁴

The confidence bands are large, but the point estimates for the contributions to the unemployment rate and inflation variances are small. It is important to acknowledge that our monetary shocks are computed after accounting, as best as we can, for time variation in the FOMC's perception about the evolution of the natural rate of unemployment, potential GDP growth, and the real rate of interest. Even if we succeed in such a task, the FOMC's assessment of these latent variables could have proven wrong. This, by all means, would represent a monetary policy shock, but it would not be accounted as such in our computations.²⁵ In this regard, our estimates may be interpreted as providing a lower-bound for the contribution of monetary policy shocks to unemployment and inflation variation. Still, one should also take into consideration that measurement error in our assessment of the FOMC's view about the latent variables could result in an overstatement of our estimated policy shock.

The contribution of policy shocks to unemployment and inflation fluctuations can also be assessed in terms of impulse-responses to a one-time realization of the shock. We compute these responses following the methodology of Romer and Romer (2004). This also provides us with an opportunity to compare their estimates with ours. Such a comparison is relevant because the methodology in Romer and Romer to evaluate the effect of the non-systematic component of monetary policy on activity and prices is also based on estimating a policy reaction function on TB forecasts of inflation, GDP growth, and the unemployment rate.²⁶ The difference is that we control for time-variation in the inflation goal, potential GDP growth, the natural rate of unemployment, and the equilibrium real federal funds rate, in addition to estimating the reaction function over different subsamples. In Romer and Romer, these variables and the reaction function parameters are taken as constant. As already mentioned, our approach could reduce the scope for monetary policy shocks if, say, a change in the FOMC's assessment of the equilibrium unemployment rate proves incorrect. Still, the reverse also holds: if the change in the assessment of the equilibrium unemployment rate is indeed correct, the Romer and Romer's strategy will count as a monetary policy shock something that is not.

²⁴ Confidence bands are computed by taking 10,000 random normal draws of coefficients that have the same variance-covariance matrix as the estimates in equation (4.1). We discard random draws that imply an unstable or indeterminate solution to the model, as this precludes computing reasonable k -step variances.

²⁵ Orphanides (2003) argues that the systematic response of policy to mismeasured activity gaps has played a crucial role in the inflation surge that occurred in the second half of the 1960s and the 1970s.

²⁶ As is well known, Romer and Romer use narrative records to infer the intended federal funds rate around FOMC meetings, while here we rely on the effective federal funds rate prevailing on average in the week after each FOMC meeting.

Following Romer and Romer, we examine the impact of monetary policy in a single-equation framework. Our regression for the unemployment rate gap takes the following form

$$\tilde{u}_t = \sum_{i=1}^4 b_{ui} MP_{t-i}^{shk} + \sum_{i=1}^4 \alpha_{ui} \tilde{u}_{t-i} + e_t^u,$$

and the one for inflation follows the same structure

$$\pi_t = \sum_{i=1}^4 b_{\pi i} MP_{t-i}^{shk} + \sum_{i=1}^4 \alpha_{\pi i} \pi_{t-i} + e_t^\pi.$$

The estimated impact of a one-time realization of our measure of the policy shock equal to 100 basis points is depicted in the next figures, together with the estimated impact from a Romer and Romer shock. As with the variance decompositions, we illustrate here results for the period 1983:Q1 to 2007:Q4, with the Romer and Romer shock series updated by Wieland and Yang (2017). The estimated responses are depicted in the two panels in Figure 10. The shock increases the unemployment rate gap by about two-tenths of one percentage point at the peak of the response. The decline in inflation over the first eight quarters averages one-tenth of a percentage point. It relevant to note that for the sample period that we consider, a 100 basis points shock amounts to about two standard deviations, and as such it represents a large shock. It is also apparent that over this period the estimated response from the Romer and Romer shocks would entail a decline in the unemployment rate, as reported also in Ramey (2016), and have little impact on average on inflation. In all, we view these findings as providing some support for our identification strategy. Together with the variance decompositions results, they suggest a limited role for discretionary monetary policy in terms of affecting output and inflation, with the caveats that we have mentioned.

5. Historical policy versus “optimal” policy

The preceding sections have attempted to carefully capture the systematic component of monetary policy, based on the historical correspondence between federal funds rate actions and Federal Reserve staff forecasts of policy goal variables. In this section, we instead ask what might an “optimal” policy have looked like? While it is of course desirable that a large component of policy be systematic and predictable, that does not imply that one could have done no better.

We begin by specifying the loss function by which policy outcomes will be judged. As is conventional in much of the literature, we posit a loss function in the squared deviations of unemployment and inflation from their desired values, along with a penalty for large changes in the federal funds rate—an interest rate smoothing motive. The horizon over which policy outcomes are considered is twenty quarters, and outcomes are discounted at the fixed rate δ , so the loss function is

$$L = \sum_{k=1}^{20} \delta^k [\omega_1 (u_{t+k} - u_{t+k}^f)^2 + \omega_2 (\pi_{t+k} - \pi_{t+k}^*)^2 + (1 - \omega_1 - \omega_2) (ff_{t+k} - ff_{t+k-1})^2] \quad (5.1)$$

The weights $[\omega_1, \omega_2, 1 - \omega_1 - \omega_2]$ are set to $[1/3, 1/3, 1/3]$. The discount rate is 0.995, or about 2% at an annual rate.

For this exercise, we assume that US policymakers choose settings of the federal funds rate so as to minimize the squared deviations of their forecasts from target or desired values. Thus we develop a model that replicates the properties of the TB forecasts, including the link from these forecasts to the federal funds rate, in order to conduct the optimal policy exercise.

The model comprises equations for the three key forecast variables (inflation over the current and next three quarters, the unemployment rate three quarters ahead, and the average growth in real GDP over the current and next three quarters), the unobservables (the natural rate, the inflation goal, potential growth and the equilibrium real rate) and the lagged real-time forecast variables. The model takes the form of reduced-form projections of the key forecast variables on real-time lagged actual data and the real federal funds rate (relative to its estimated equilibrium level); a policy rule of the form used in the empirical work above; and reduced-form equations for the lagged real-time data:

$$\begin{aligned}
\pi_t^{4q,f} - \pi_t^* &= \sum_{i=1}^2 \left\{ a_i^\pi (\pi_{t-i} - \pi_{t-i}^*) + b_i^\pi (u_{t-i} - u_{t-i}^*) + c_i^\pi (\Delta y_{t-i} - \Delta y_{t-i}^*) \right\} + d^\pi (\bar{ff}_{t-1} - \bar{\pi}_{t-1,t+3}^{4,f} - \bar{r}_{t-1}^*) \\
u_t^{q4,f} - u_t^* &= \sum_{i=1}^2 \left\{ a_i^U (\pi_{t-i} - \pi_{t-i}^*) + b_i^U (u_{t-i} - u_{t-i}^*) + c_i^U (\Delta y_{t-i} - \Delta y_{t-i}^*) \right\} + d^U (\bar{ff}_{t-1} - \bar{\pi}_{t-1,t+3}^{4,f} - \bar{r}_{t-1}^*) \\
\Delta y_t^{4q,f} - \Delta y_t^* &= \sum_{i=1}^2 \left\{ a_i^Y (\pi_{t-i} - \pi_{t-i}^*) + b_i^Y (u_{t-i} - u_{t-i}^*) + c_i^Y (\Delta y_{t-i} - \Delta y_{t-i}^*) \right\} + d^Y (\bar{ff}_{t-1} - \bar{\pi}_{t-1,t+3}^{4,f} - \bar{r}_{t-1}^*) \\
ff_t &= \rho_1 ff_{t-1} + \rho_2 ff_{t-2} + (1 - \rho_1 - \rho_2) [r_t^* + \pi_t^* + \alpha_\pi (\pi_{t,t+4}^{4,f} - \pi_t^*) + \alpha_U (u_{t,t+4}^f - u_t^*) + \alpha_{dY} (\Delta y_{t,t+4}^{4,f} - \Delta y_t^*)] + \sum_{j=0}^{20} \nu_{t-j} \\
\bar{ff}_t &\equiv 0.25 (ff_t + ff_{t-1} + ff_{t-2} + ff_{t-3}) \\
\begin{bmatrix} \pi_{t-1} - \pi_{t-1}^* \\ u_{t-1} - u_{t-1}^* \\ \Delta y_{t-1} - \Delta y_{t-1}^* \end{bmatrix} &= \beta_1 \begin{bmatrix} \pi_{t-2} - \pi_{t-2}^* \\ u_{t-2} - u_{t-2}^* \\ \Delta y_{t-2} - \Delta y_{t-2}^* \end{bmatrix} + \beta_2 \begin{bmatrix} \pi_{t-3} - \pi_{t-3}^* \\ u_{t-3} - u_{t-3}^* \\ \Delta y_{t-3} - \Delta y_{t-3}^* \end{bmatrix} + \begin{bmatrix} c_\pi \\ c_U \\ c_Y \end{bmatrix} \\
\begin{bmatrix} \pi_t^* \\ u_t^* \\ \Delta y_t^* \\ r_t^* \end{bmatrix} &= \begin{bmatrix} \pi_{t-1}^* \\ u_{t-1}^* \\ \Delta y_{t-1}^* \\ r_{t-1}^* \end{bmatrix} + \begin{bmatrix} e_t^\pi \\ e_t^U \\ e_t^Y \\ e_t^r \end{bmatrix}
\end{aligned} \tag{5.2}$$

The shocks $\sum_{j=0}^{20} \nu_{t-j}$ are simply a mechanism for computing an optimal path for the federal funds

rate. They are the period-by-period deviations from the policy rule, and because they enter with a lag they are known in advance by the agents in the model, so that they feed into the expectations of the key variables given by the first three equations in the system. We estimate the model parameters via full-information maximum likelihood. As a test of its ability to replicate the dynamics of the TB forecasts, the next figure shows fitted values over the estimation sample.

We perform a particular optimal policy exercise: Starting from a point in the sample, we take the lagged funds rate and forecast data as given, and then choose a sequence of shocks $\sum_{j=0}^{20} \nu_{t-j}$ that minimizes the loss function (5.1). The optimization is re-initialized at each period, so that optimized funds rate settings do not feed into the solution for the forecasts in subsequent periods; they only feed into the twenty-quarter sequence of simulated forecasts that are relevant for the optimal funds rate in that period.

Figure 11 displays the results of this exercise. The discrepancy between the optimized federal funds rate and the actual funds rate is relatively small for any period—typically 50 basis points or less. Notably, the average discrepancy between the two is negative 0.24 percentage points, which indicates that on average, according to this metric, policy has been a bit tighter (interest rates were a bit higher) than the optimal policy would suggest.

Figure 12 compares the funds rate path computed above with various versions of the Taylor (1993) rule. The rule must be modified somewhat from the published rule, as the inflation rate that the Fed follows has changed over time, and data have been revised. The variants use the real-time data for inflation and unemployment as recorded in TB dataset, adjust the coefficient on the unemployment gap to one versus the one-half coefficient on the output gap in the original article, and (in some variants) allow for time-variation in the real rate and the inflation goal. In all variants, the natural rate of unemployment is allowed to vary, using the estimates from section 3 above. A final variant includes interest rate smoothing, using a coefficient of 0.85, consistent with the estimates presented above. Thus the rules considered are

$$\begin{aligned}
ff_t &= 0.25[\pi_{t-1}^f + \pi_{t-2}^f + \pi_{t-3}^f + \pi_{t-4}^f] + (u_{t-1}^f - u_{t-1}^*) + 0.5(\pi_{t-1}^f - 2) + 2 \\
ff_t &= 0.25[\pi_{t-1}^f + \pi_{t-2}^f + \pi_{t-3}^f + \pi_{t-4}^f] + (u_{t-1}^f - u_{t-1}^*) + 0.5(\pi_{t-1}^f - 2) + r_t^* \\
ff_t &= 0.25[\pi_{t-1}^f + \pi_{t-2}^f + \pi_{t-3}^f + \pi_{t-4}^f] + (u_{t-1}^f - u_{t-1}^*) + 0.5(\pi_{t-1}^f - \pi_{t-1}^*) + r_t^* \\
ff_t &= 0.85ff_{t-1} + 0.15\{0.25[\pi_{t-1}^f + \pi_{t-2}^f + \pi_{t-3}^f + \pi_{t-4}^f] + (u_{t-1}^f - u_{t-1}^*) + 0.5(\pi_{t-1}^f - \pi_{t-1}^*) + r_t^*\}
\end{aligned} \tag{5.3}$$

Without interest rate smoothing, the prescriptions of these rules lie quite far from either the optimal or the actual federal funds rate. Of course, adding rate smoothing implies that the funds rate in any period will not deviate too far from the previous observation, so this rule's prescriptions (show in the red dot line) are closer to the actual and the optimal policy paths.

6. Implications for the Great Recession and recovery period

Arguably, the paper has focused on the relatively tranquil period from 1983-2007, the “Great Moderation.” Policy may have been conducted reasonably well during this period, and some have argued that monetary policy is largely responsible for the favorable outcomes that occurred during that time.

That is of course less true for the period after 2007. It may be of interest to see what the estimated policy rule (along with the unobservables) imply for the conduct of policy during the Great Recession and recovery to date. What would the estimated Taylor rule from Table 2 imply for the funds rate during this period?

To answer this question, we use the model from section 5, imposing the estimated coefficients on the policy rule from Table 2, and feeding in the non-monetary policy shocks for the unemployment, inflation and real growth forecasts. This allows the simulated path of the funds rate to affect the economy, given the other shocks that are estimated to have buffeted the economy during that period. For the unobservables, we use the TB estimates of potential growth and the natural rate as published in (or inferred from) the TB. The inflation goal is assumed to be two percent, consistent with the announced goal as of January 2012 and also consistent with the long-run expectations for the PCE as measured by the Survey of Professional Forecasters, a measurement variable employed in the state-space system described in section 2. For the equilibrium real federal funds rate, we employ a few different assumptions that allow the rate to decline geometrically from its last estimated value in 2007:Q4 to either 1 percent, 0.5 percent, or zero.

Importantly, we have no direct way of incorporating the balance sheet actions of the Fed during this period. However, the forecasts incorporate the staff's estimated effects of these policies on real variables and inflation, and the model will capture some of that dynamic. The results of this exercise are shown in Figure 13. As the figure indicates, absent the zero (or effective zero) lower bound, the funds rate would have been lowered to -3 or -4 percent, depending on the estimate of the equilibrium real federal funds rate. With the imposition of the zero lower bound, the path of the funds rate differs relatively little from the realized funds rate. Under the assumption that the equilibrium real rate is one percent (the blue line), "lift-off" occurs several quarters earlier than the actual case. But all in all, this exercise suggests that the estimated policy rule, which was shown to be near-optimal prior to the Great Recession, accurately captures the response of policymakers during the Great Recession as well.

7. Conclusions

As suggested in the introduction, policy rules occupy an important place in the conduct of contemporary monetary policy. Rules attempt to capture the systematic component of monetary policy, and when successful in doing so, they provide important benchmarks that central banks do well to use in deciding the appropriate setting of their instrument. Rules also allow central banks and academics to endogenize monetary policy in macroeconomic models, a marked advance over earlier practice. This advantage is of far more than academic interest, as such models allow central banks to conduct counterfactual policy exercises, perform optimal policy exercises, and forecast the economy in real time, all of which require a reasonable representation of endogenous monetary policy.

However, the characterization of the systematic component of monetary policy can only occur if one knows key aspects of the economy in which the central bank operates. While knowledge of the central bank's inflation goal was the subject of inference in an earlier time, most central banks now espouse explicit numerical objectives for their inflation goals. However, none possess knowledge of the current values of the natural rate of unemployment, the equilibrium real rate of interest, or the growth rate or level of equilibrium output. All must be inferred indirectly from data, jointly with restrictions that impose some degree of economic structure on the data. Thus all are subject to considerable uncertainty. As a consequence, one must acknowledge that even if one takes the rest of a policy rule as gospel, the implementation of the rule and its implications are subject to considerable uncertainty. This caveat applies to non-rule-based monetary policy as well, and suggests

that a portfolio approach to policy is likely to work best, in which policymakers balance the guidance provided by rules, optimal policy exercises, counterfactual projections, and judgment.

But it does not seem reasonable to take any rule as gospel. In general, the optimal policy response need not be well-described by a linear rule that is a function of a restricted number of variables and their lags. And within the class of such rules, one cannot know *a priori* whether a rule with either imposed or estimated coefficients will produce the best outcomes for the economy.

Thus we conclude that while policy rules are an essential part of the central banker's toolkit, they need to be used judiciously—that is, along with good judgment. Systematic policy is a goal and a virtue, but simple rules need no more yield the best systematic policy than simple aphorisms—“if it ain't broke, don't fix it”—yield the happiest lives.

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Table 1			
Policy rule estimates, 1983:1-2007:4			
Variable	Coefficient	Standard error	p-value
$\hat{f}f_{t-1}$	1.12	0.062	0.0000
$\hat{f}f_{t-2}$	-0.23	0.065	0.0005
$\pi_{t,t+4}^{4,f} - \pi_t^*$	2.37	1.48	0.1102
$u_{t,t+4}^f - u_t^*$	-3.00	1.37	0.0288
$\Delta y_{t,t+4}^{4,f} - \Delta y_t^*$	2.34	1.05	0.0261
Log likelihood: -241.0618 Akaike info criterion: 5.041236 Schwarz criterion: 5.327805			
Policy rule estimates, 1969:1-1979:3			
$\hat{f}f_{t-1}$	0.48	0.22	0.0283
$\hat{f}f_{t-2}$	0.00	0.22	0.9961
$\pi_{t,t+4}^{4,f} - \pi_t^*$	1.83	0.33	0.0000
$u_{t,t+4}^f - u_t^*$	-2.37	0.84	0.0048
Log likelihood: -182.94 Akaike info criterion: 9.3837 Schwarz criterion: 9.1252			

Table 2 Instrumented estimates of policy parameters 1983:1-2007:4 Method 1: System state-space estimates			
Variable	Coefficient	Standard error (corrected)	<i>p</i> -value
ff_{t-1}	1.14	0.071	0.0000
ff_{t-2}	-0.27	0.080	0.0011
$\pi_{t,t+4}^{4,f} - \pi_t^*$	2.64	1.76	0.135
$u_{t,t+4}^f - u_t^*$	-2.30	1.00	0.109
$\Delta y_{t,t+4}^{4,f} - \Delta y_t^*$	1.62	1.15	0.0492
Log likelihood: -243.57 Akaike info criterion: 5.091 Schwarz criterion: 5.378			
Method 2: GMM			
ff_{t-1}	1.05	0.071	0.0000
ff_{t-2}	-0.20	0.064	0.0029
$\pi_{t,t+4}^{4,f} - \pi_t^*$	2.92	1.02	0.0051
$u_{t,t+4}^f - u_t^*$	-2.90	0.47	0.0000
$\Delta y_{t,t+4}^{4,f} - \Delta y_t^*$	2.45	0.69	0.0006
Adjusted R-squared: 0.959 J-statistic: 12.53 (<i>p</i> -value = 0.638) Standard error: 0.492			
Method 2: GMM 1969:1-1979:3			
ff_{t-1}	0.59	0.062	0.0000
ff_{t-2}	-0.017	0.048	0.7281
$\pi_{t,t+4}^{4,f} - \pi_t^*$	1.43	0.039	0.0000
$u_{t,t+4}^f - u_t^*$	-2.35	0.15	0.0000
Adjusted R-squared: 0.879 J-statistic: 9.77 (<i>p</i> -value = 0.878) Standard error: 0.793			

Table 3 The relative importance of lagged real-time data versus forecasts in policy rule estimates 1983:1-2007:4			
Variable	Coefficient	Standard error	<i>p</i> -value
ff_{t-1}	1.05	0.064	0.0000
ff_{t-2}	-0.19	0.067	0.0064
$\pi_{t,t+4}^{4,f} - \pi_t^*$	2.84	1.34	0.0362
$u_{t,t+4}^f - u_t^*$	-1.52	0.81	0.0653
$\Delta y_{t,t+4}^{4,f} - \Delta y_t^*$	0.60	0.74	0.422
$\Delta y_{t-1} - \Delta y_t^*$	0.81	0.27	0.0036
Log likelihood: -235.43 Akaike info criterion: 4.95 Schwarz criterion: 5.26			
Memo: LR test that all six lags jointly = 0 (<i>p</i> -value): 0.168			

Table 3a The relative importance of lagged real-time data versus forecasts in policy rule estimates 1969:1-1979:3			
Variable	Coefficient	Standard error	<i>p</i> -value
ff_{t-1}	0.51	0.060	0.0000
ff_{t-2}	-0.28	0.065	0.0001
$\pi_{t,t+4}^{4,f} - \pi_t^*$	0.82	0.051	0.0000
$u_{t,t+4}^f - u_t^*$	-0.91	0.6	0.0014
$\pi_{t-2} - \pi_t^*$	0.31	0.041	0.0000
$u_{t-1} - u_t^*$	-0.68	0.14	0.0000
R-squared: 0.918 J-statistic: 9.83, <i>p</i> -value = 0.775 Standard error: 0.653			

Table 4			
Estimated policy rule with additional data from very large dataset 1983:Q1-2007:Q4			
Variable	Coefficient	Standard error	p-value
ff_{t-1}	0.83	0.020	0.0000
$\pi_{t,t+4}^{4,f} - \pi_t^*$	1.77	0.53	0.0013
$u_{t,t+4}^f - u_t^*$	-1.46	0.30	0.0000
$\Delta y_{t,t+4}^{4,f} - \Delta y_t^*$	0.89	0.34	0.0118
1 st PC, real variables	0.29	0.052	0.0000
2 nd PC, financial “stock” variables	0.27	0.044	0.0000
1 st PC, wage and price variables	0.19	0.070	0.0083
R-squared: 0.965 Adjusted R-squared: 0.967 S.E. of regression: 0.440 J-statistic (p-value): 17.17 (0.80)			

Figure 1

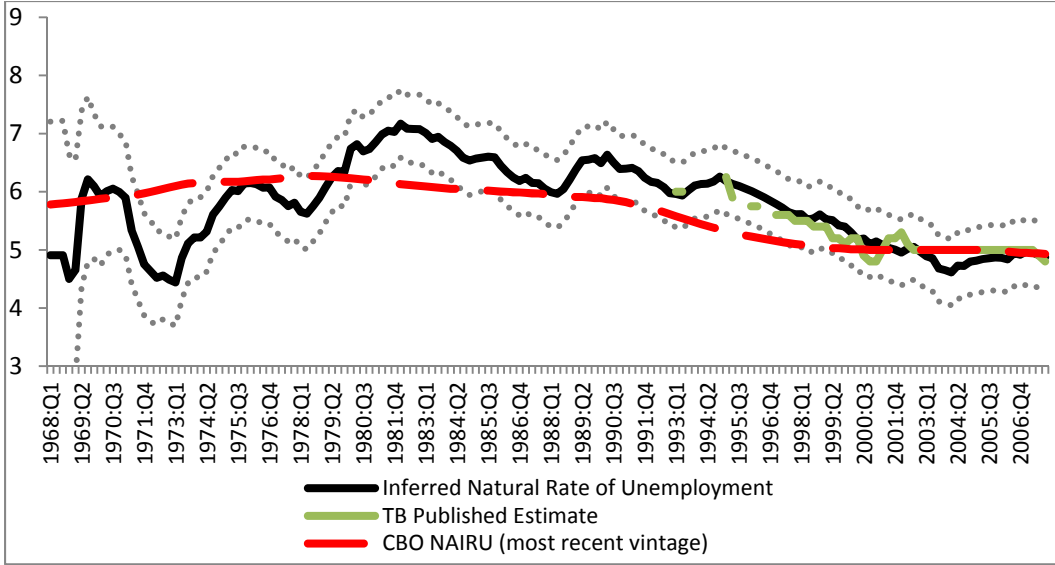


Figure 2

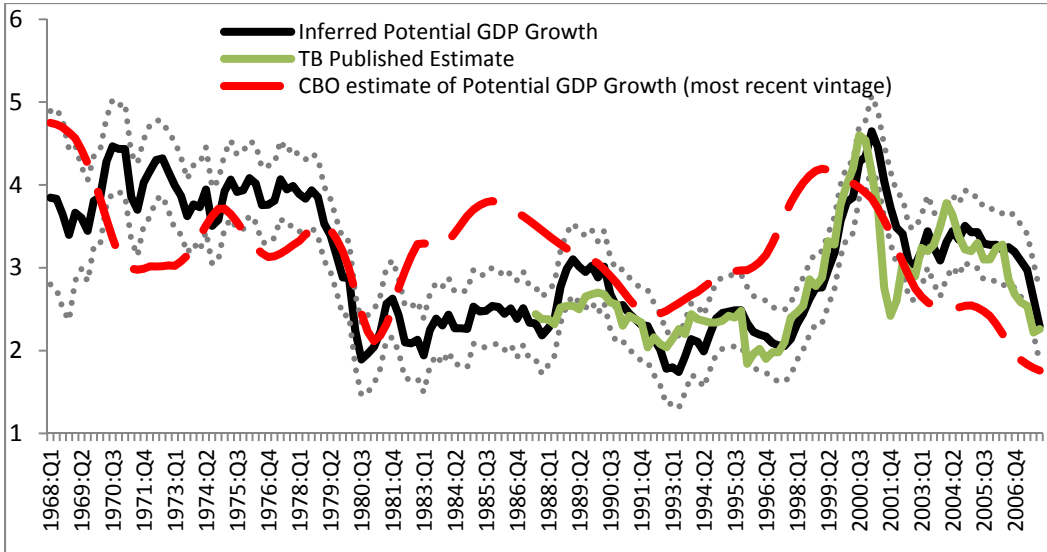


Figure 3

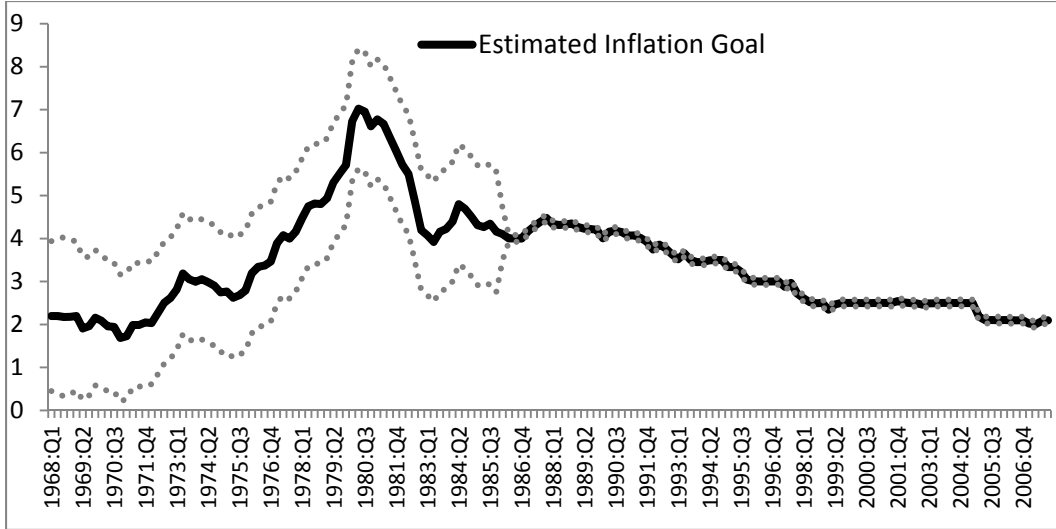


Figure 4

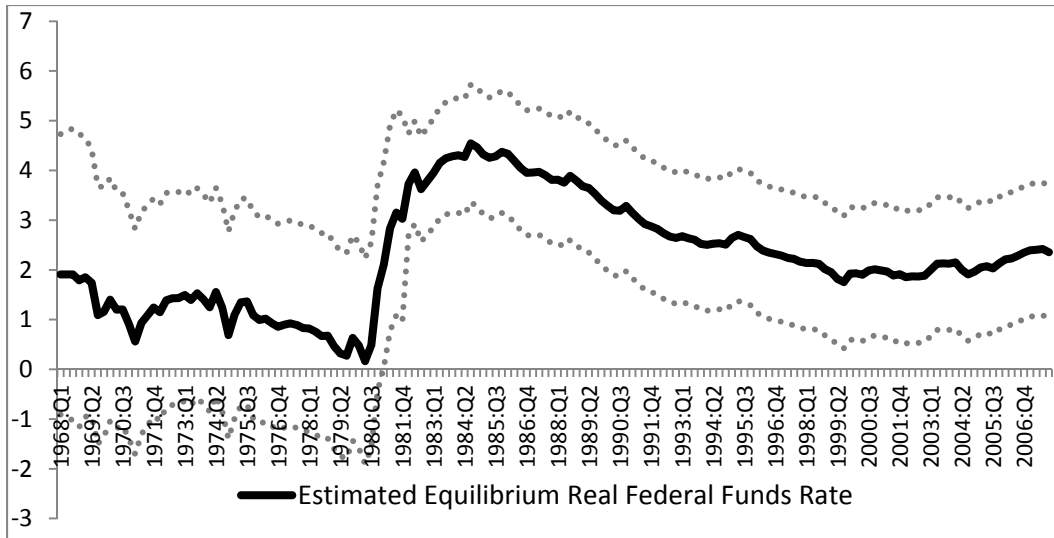


Figure 5

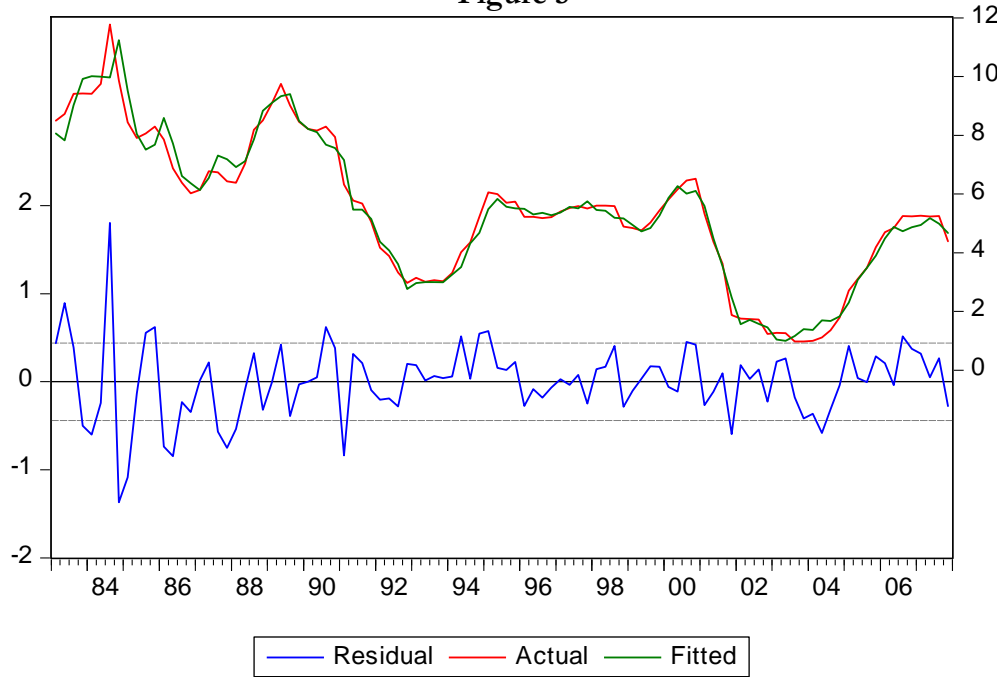


Figure 6

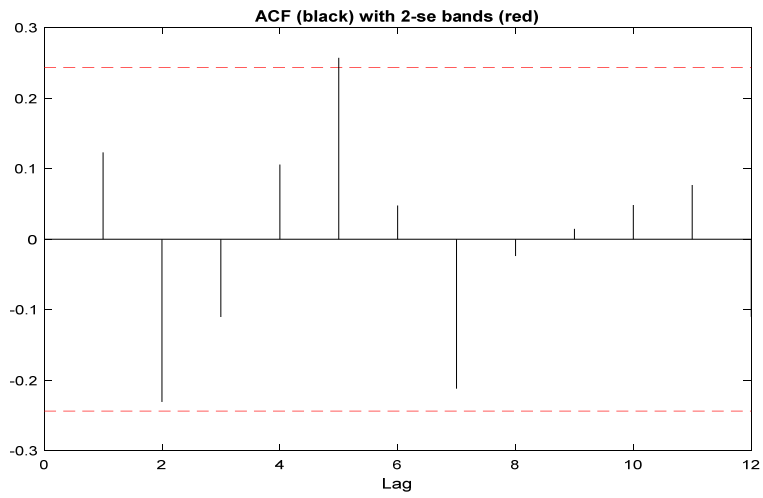


Figure 7

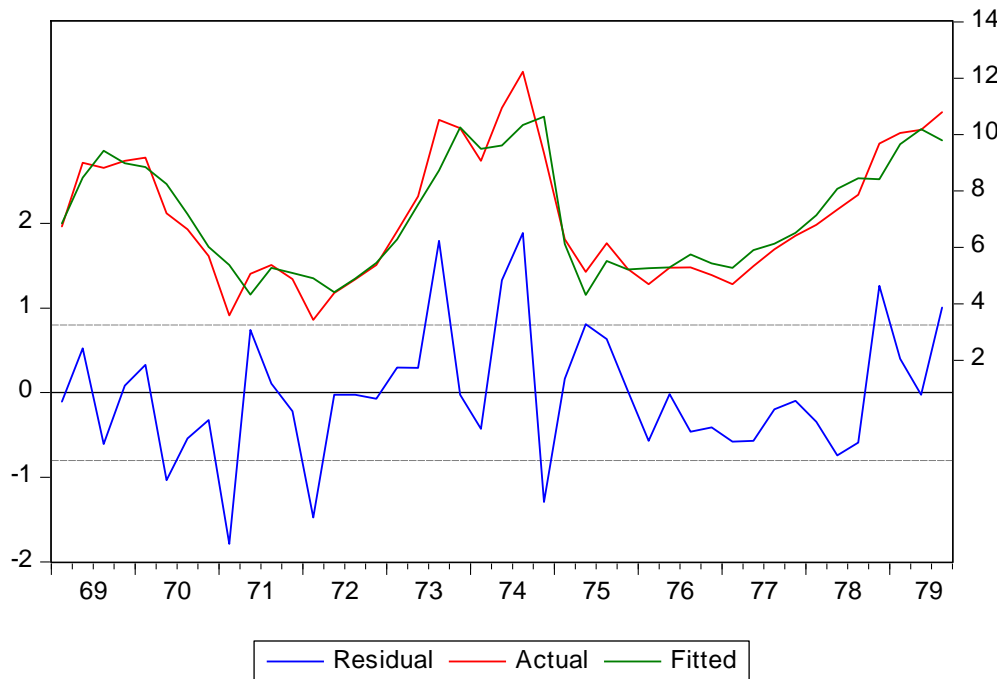


Figure 8

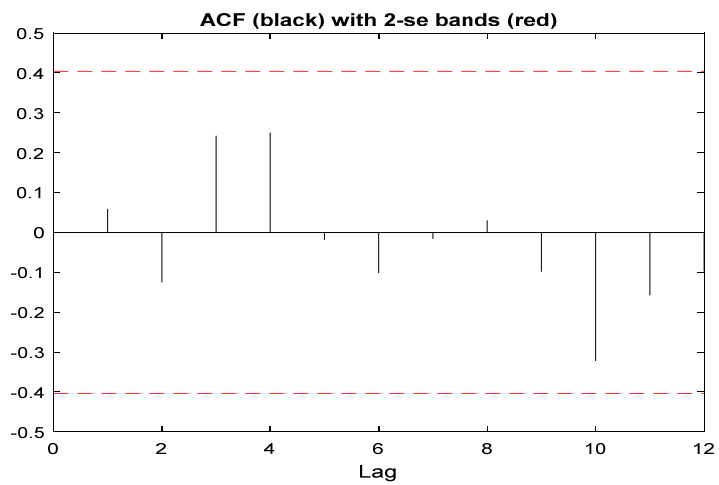


Figure 9

Variance attributable to monetary policy shock

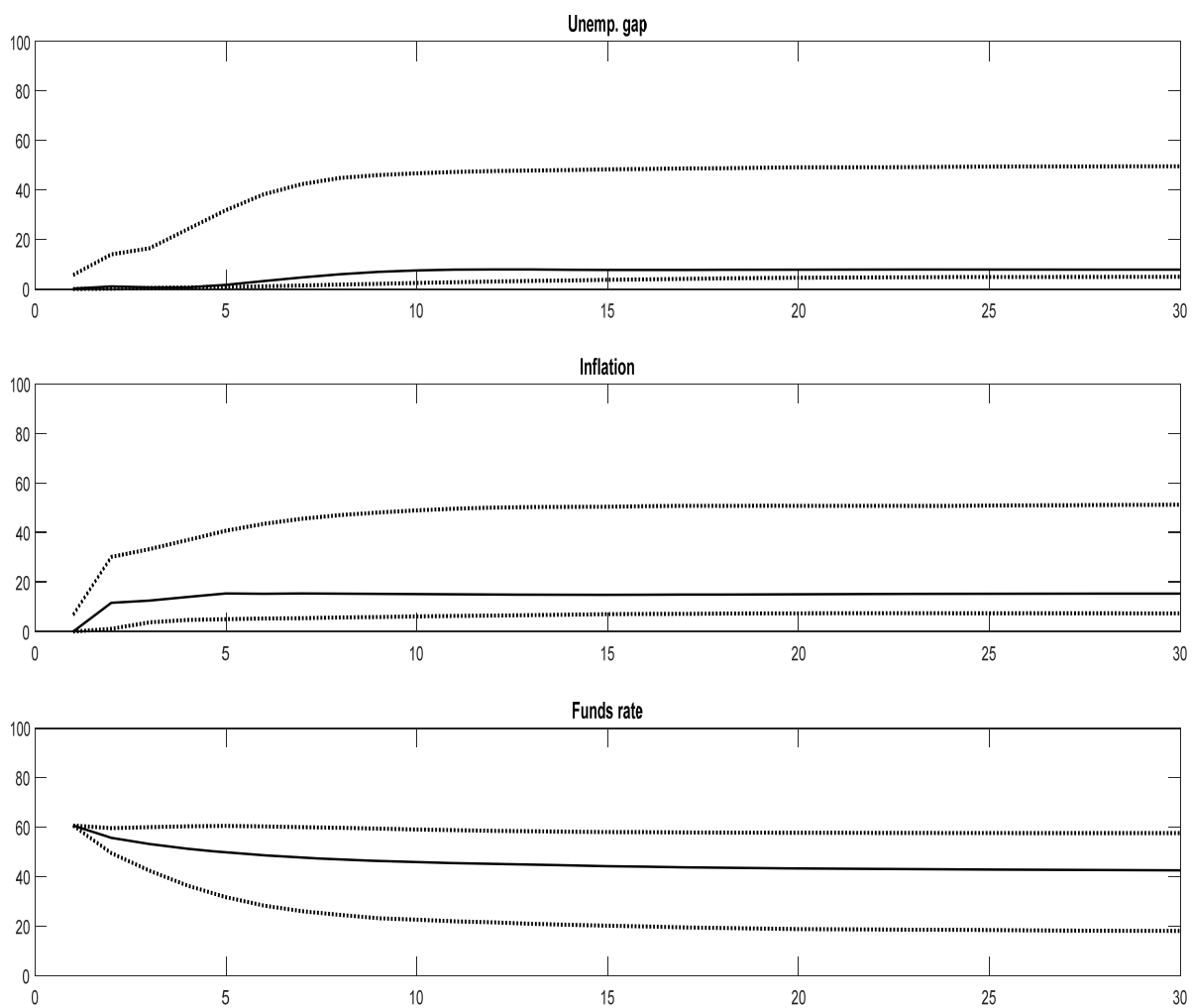


Figure 10

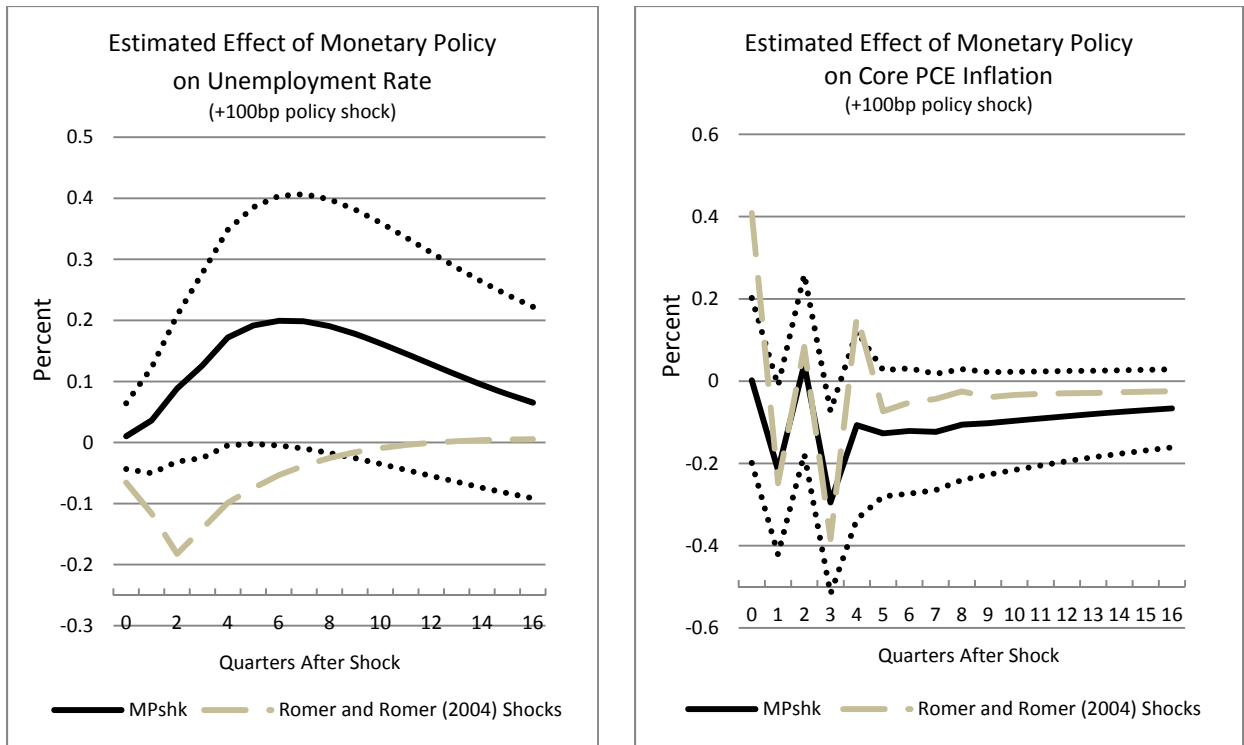


Figure 11

Optimal forecast-based policy, period-by-period

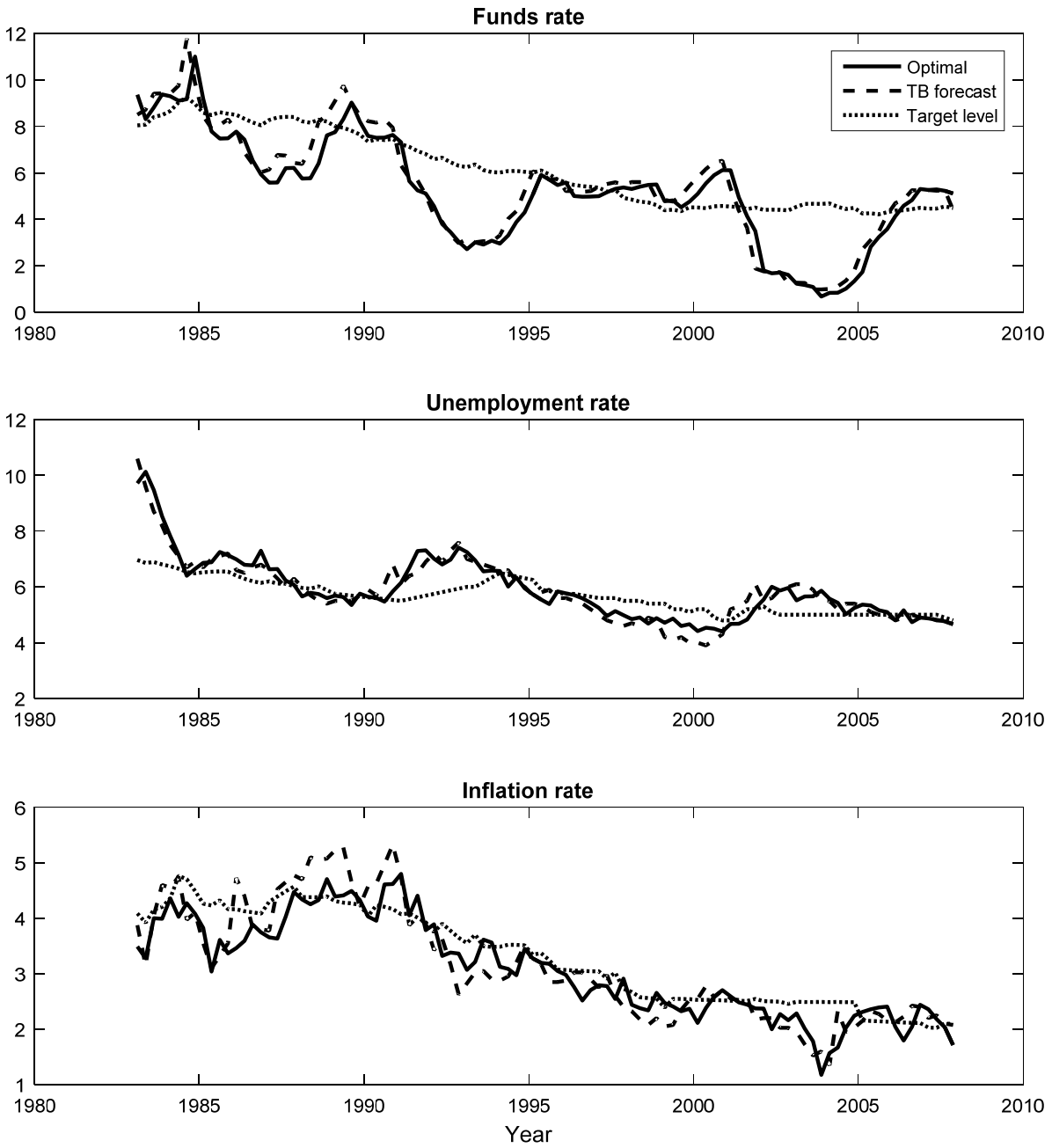


Figure 12
Optimal funds rate versus various Taylor rules

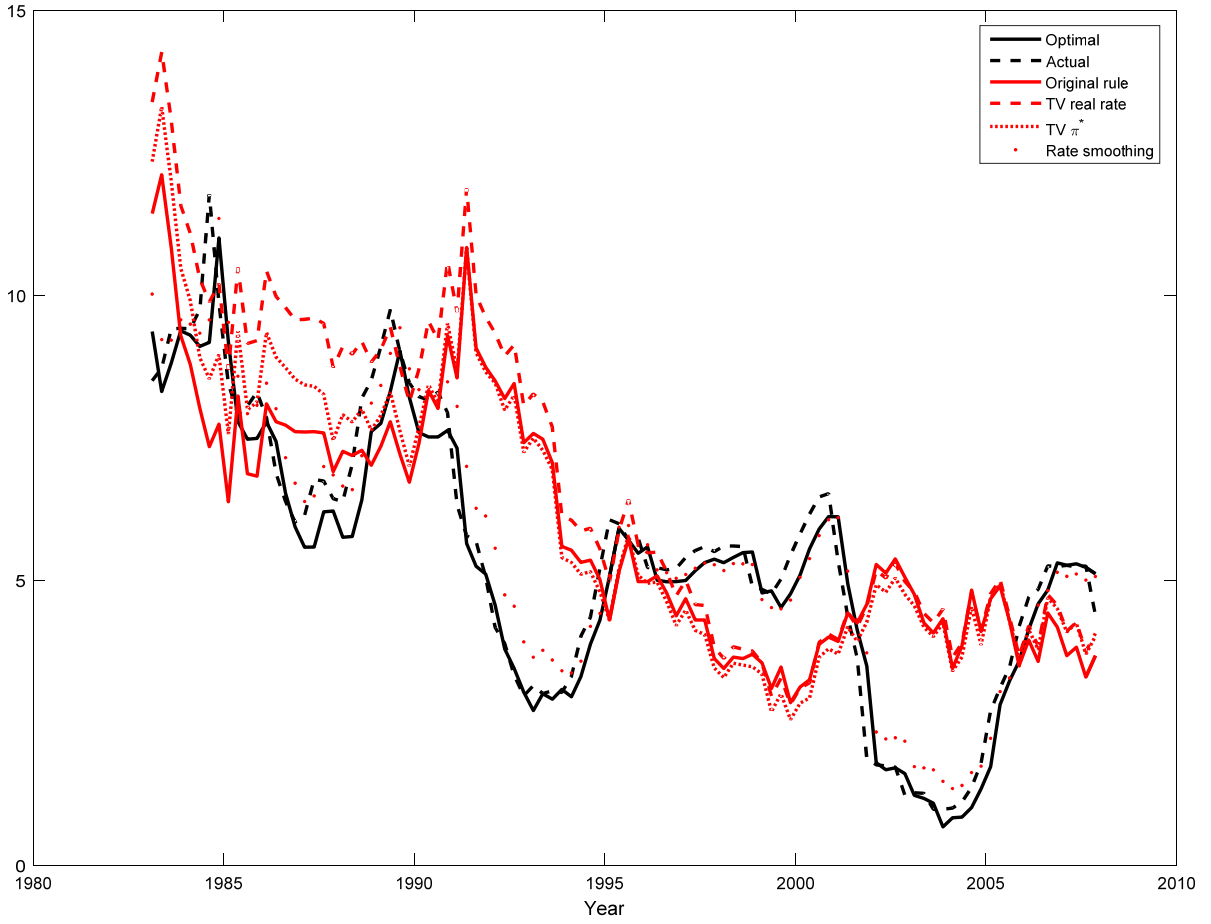
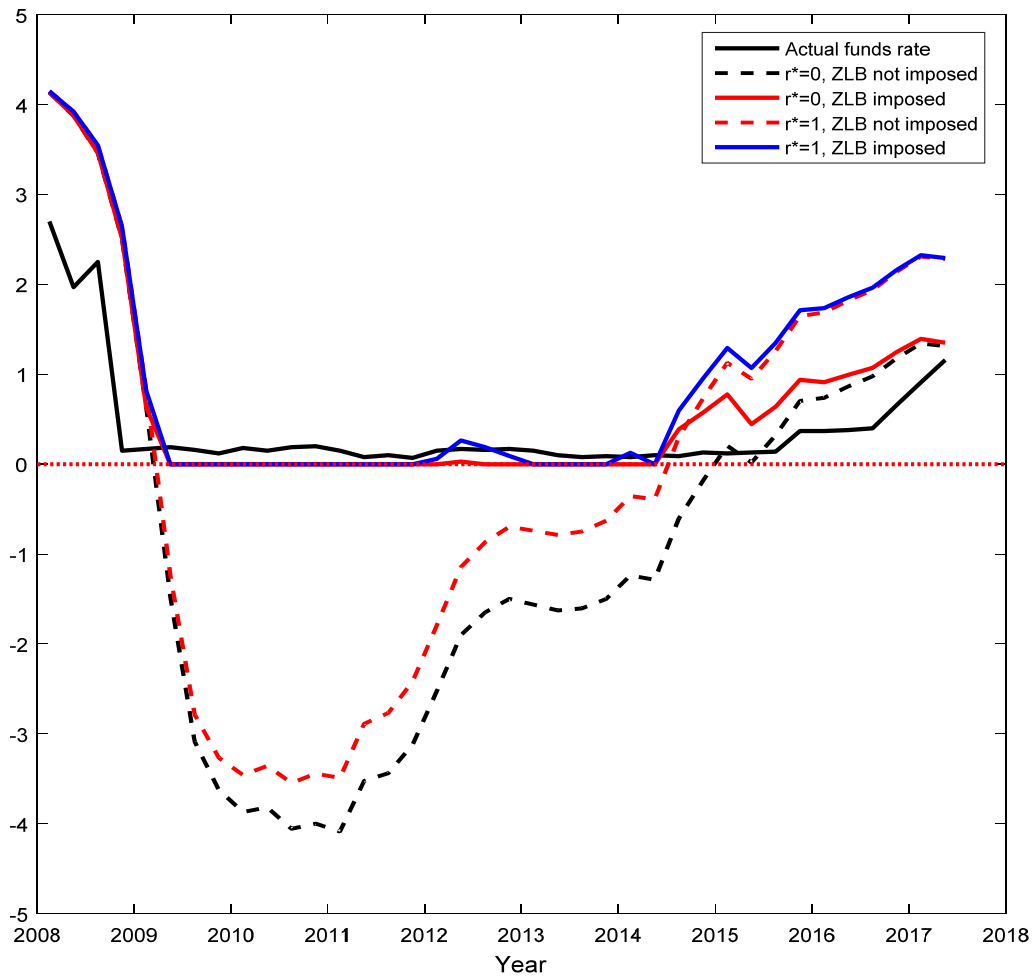


Figure 13

Policy rule simulated with optimal policy model, policy shocks = 0



Appendix 1 Data

Greenbook/Tealbook data

- Work with quarterly dataset
- Select one forecast per quarter, so that (a) current-quarter forecast = current calendar quarter (some forecasts have current quarter = last quarter), and (b) horizon for forecasts is maximized (at least four quarters).
- Focus on inflation, unemployment and real GDP growth
- Have observations on the federal funds rate—the average of the three days following the FOMC date, as well as lagged federal funds rate, defined as the average value prevailing in the week after the FOMC meeting.
- The definition of inflation varies over the sample: first GDP deflator, then core CPI, then core PCE, with breakpoints in 1985, 2005.

Principal components of macroeconomic and financial data series

We compute principal components from a large set of macroeconomic and financial data. The table below lists all of the variables used. We group the principal components into five categories: (i) real activity data, (ii) wage and price data, (iii) interest rates, (iv) credit variables, (v) equities, exchange rates and term and risk premia. Before computing the principal component, we transform the series to make them stationary when needed. The variables are listed below together with their mnemonic.

Mnemonic	Definition
GDPPLUS	US GDPplus [Alternate Measure of Q/Q Rate of Growth of Real GDP] (SAAR, %Chg)
CUMFG	Capacity Utilization: Manufacturing [SIC] (SA, Percent of Capacity)
NAPMC	ISM Mfg: PMI Composite Index (SA, 50+ = Econ Expand)
NAPMOI	ISM Mfg: Production Index (SA, 50+ = Econ Expand)
YPSVR	Personal Saving Rate (SA, %)
PTVH	Change in Private Inventories: Contribution to Real GDP %Chg (SAAR, %Pt)
HSM	Manufacturers' Shipments of Mobile Homes (SAAR, Thous.Units)
GDPH	Real Gross Domestic Product (SAAR, Bil.Chn.2009\$)
GDYH	Real Gross Domestic Income (SAAR, Bil.Chn.2009\$)
GDPPOTHQ	Real Potential Gross Domestic Product [CBO] (SAAR, Bil.Chn.2009\$)
IP	Industrial Production Index (SA, 2007=100)
LXNFA	Nonfarm Business Sector: Real Output Per Hour of All Persons (SA, 2009=100)
LXNCA	Nonfinancial Corporations: Real Output Per Hour, All Employees (SA,2009=100)
CH	Real Personal Consumption Expenditures (SAAR, Bil.Chn.2009\$)
CSH	Real Personal Consumption Expenditures: Services (SAAR, Bil.Chn.2009\$)
CNH	Real Personal Consumption Expenditures: Nondurable Goods (SAAR, Bil.Chn.2009\$)
CDH	Real Personal Consumption Expenditures: Durable Goods (SAAR, Bil.Chn.2009\$)
CDVH	Real PCE: Motor Vehicles & Parts (SAAR, Bil.Chn.2009\$)
YPMH	Real Personal Income (SAAR, Bil.Chn.2009\$)

YPXTPH	Real Personal Income excluding Current Transfer Receipts (SAAR, Bil.Chn.2009\$)
YPWH	Wages & Salaries (SAAR, Bil. 2009\$)
YCOMPRIH	Compensation of Employees (SAAR, Bil. 2009\$)
YPDH	Real Disposable Personal Income (SAAR, Bil.Chn.2009\$)
Y_PRIV	Real Personal Income Ex-Govt. Transfers
FNEH	Real Private Nonresidential Fixed Investment: Equipment (SAAR, Bil.Chn.2009\$)
FNENH	Real Pvt Nonres Fixed Investment: Info Processing Eqpt (SAAR, Bil.Chn.2009\$)
FNEIH	Real Private Fixed Investment: Industrial Equipment (SAAR, Bil.Chn.2009\$)
FNETH	Real Private Fixed Investment: Transportation Equipment (SAAR, Bil.Chn.2009\$)
FNEOH	Real Private Fixed Invest: Other Equipment (SAAR, Bil.Chn.2009\$)
FNPH	Real Pvt Nonres Investment: Intellectual Property Products (SAAR, Bil.Chn.2009\$)
FNSH	Real Private Nonresidential Fixed Investment: Structures (SAAR, Bil.Chn.2009\$)
FRH	Real Private Residential Fixed Investment (SAAR, Bil.Chn.2009\$)
LRGAP	RA16-NAIRUQ
LUMD	Median Duration of Unemployment (SA, Weeks)
LUAD	Average [Mean] Duration of Unemployment (SA, Weeks)
LU0P	Unemployed for Less Than 5 Weeks: % of Civilians Unemployed (SA, %)
LU5P	Unemployed for 5-14 Weeks: % of Civilians Unemployed (SA, %)
LU15P	Unemployed for 15-26 Weeks: % of Civilians Unemployed (SA, %)
LUT27P	Unemployed for 27 Weeks and Over: % of Civilians Unemployed (SA, %)
NAPMEI	ISM Mfg: Employment Index (SA, 50+ = Econ Expand)
HWI	Help-Wanted Index (Barnichon)
RA16	Unemployment Rate (SA, %)
NAIRUQ	Natural Rate of Unemployment [CBO] (%)
RA15	Unemployment Rate: Unemployed 15 Weeks & Over [% of Civilian Labor Force](SA, %)
RA27	Unemployment Rate: Unemployed Less Than 27 Weeks (SA, %)
LICM	Initial Claims for Unemployment Insurance, State Programs, Wkly Avg (SA, Thous)
EA16	Civilian Employment (SA, Thous)
QA16	Employment-Population Ratio (SA, %)
FA16	Labor Force Participation Rate (SA, %)
LANAGRA	All Employees: Total Nonfarm (SA, Thous)
LAPRIVA	All Employees: Total Private Industries (SA, Thous)
LAGOVTA	All Employees: Government (SA, Thous)
LRPRIVA	Average Weekly Hours: Prod & Nonsupervisory: Private Industries (SA, Hrs)
LRGOODA	Avg Weekly Hours: Prod & Nonsupervisory: Goods-producing Industries (SA, Hrs)
LRPSRVA	Avg Wkly Hrs: Prod & Nonsupervisory: Pvt Service-providing Industries (SA, Hrs)
LOMANUA	Average Weekly Hours: Prod & Nonsupervisory: Overtime: Manufacturing (SA, Hrs)
LHTNAGRA	Aggregate Hours: Nonfarm Payrolls, Total (SAAR, Bil.Hrs)
JGDP	Gross Domestic Product: Chain Price Index (SA, 2009=100)
JC	Personal Consumption Expenditures: Chain Price Index (SA, 2009=100)
JCXFE	PCE less Food & Energy: Chain Price Index (SA, 2009=100)
JCGSE	PCE: Energy Goods & Services: Chain Price Index (SA, 2009=100)
JCNFO	PCE: Food & Bev Purch for Off-Premises Consumptn: Chain Price Idx(SA, 2009=100)

JCXEGM	PCE excluding Energy Goods & Services: Chain Price Index (SA, 2009=100)
JCN	PCE: Nondurable Goods: Chain Price Index (SA, 2009=100)
JCD	PCE: Durable Goods: Chain Price Index (SA, 2009=100)
JCS	Personal Consumption Expenditures: Services: Chain Price Index (SA, 2009=100)
PCU	CPI-U: All Items (SA, 1982-84=100)
PCUSLFE	CPI-U: All Items Less Food and Energy (SA, 1982-84=100)
PCUSLE	CPI-U: All Items Less Energy (SA, 1982-84=100)
PCUSLF	CPI-U: All Items Less Food (SA, 1982-84=100)
PCUSLS	CPI-U: All Items Less Shelter (SA, 1982-84=100)
PCUSLM	CPI-U: All Items Less Medical Care (SA, 1982-84=100)
PCUCC	CPI-U: Commodities (SA, 1982-84=100)
PCUCS	CPI-U: Services (SA, 1982-84=100)
PCUCCDN	CPI-U: Durables (NSA, 1982-84=100)
PCUSND	CPI-U: Nondurables (SA, 1982-84=100)
UAXAF	CPI-U: Apparel Less Footwear (SA, 1982-84=100)
PCUT	CPI-U: Transportation (SA, 1982-84=100)
PCUM	CPI-U: Medical Care (SA, 1982-84=100)
PZALL	KR-CRB Spot Commodity Price Index: All Commodities (1967=100)
PZTEXP	Spot Oil Price: West Texas Intermediate [Prior'82=Posted Price] (\$/Barrel)
SP3000	PPI: Finished Goods (SA, 1982=100)
SP2000	PPI: Intermediate Materials, Supplies and Components (SA, 1982=100)
SP1000	PPI: Crude Materials for Further Processing (SA, 1982=100)
SP3100	PPI: Finished Consumer Goods (SA, 1982=100)
LEPRIVA	Avg Hourly Earnings: Prod & Nonsupervisory: Total Private Industries(SA, \$/Hour)
LEGOODA	Avg Hourly Earnings: Prod & Nonsupervisory: Goods-producing Industries(SA, \$/Hr)
LEPSRVA	Avg Hrly Earn: Prod & Nonsupervisory: Private Svc-providing Industries(SA, \$/Hr)
LXNFC	Nonfarm Business Sector: Compensation Per Hour (SA, 2009=100)
LXNCC	Nonfinancial Corporations: Compensation per Hour (SA, 2009=100)
LSP	ECI: Compensation: Private Industry Workers (SA, Dec-05=100)
LXNFBL	Nonfarm Business: Labor Share, All Persons (SA)
LXNCBL	Nonfinancial Corporations: Labor Share, All Employees (SA)
CESENT	University of Michigan: Consumer Sentiment (NSA, Q1-66=100)
CEXP	University of Michigan: Consumer Expectations (NSA, Q1-66=100)
CCIN	Conference Board: Consumer Confidence (SA, 1985=100)
CCIEN	Conference Board: Consumer Expectations (SA, 1985=100)
HST	Housing Starts (SAAR, Thous.Units)
HPT	New Pvt Housing Units Authorized by Building Permit (SAAR, Thous.Units)
SDY5COMM	S&P: Composite 500, Dividend Yield (%)
SPE5COMM	S&P: 500 Composite, Price/Earnings Ratio (Ratio)
SPECAPE	Shiller Cyclically Adjusted S&P Price to Earnings Ratio (Ratio)
REQ	Real expected rate of return on equity
PL10COG6	Nonfinancial Corporate Business: Market Value of Equities/Net Worth (%)
SPNY	Stock Price Index: NYSE Composite (Avg, Dec-31-02=5000)

SP500	Stock Price Index: Standard & Poor's 500 Composite (1941-43=10)
SPSPI	Stock Price Index: Standard & Poor's 500 Industrials (1941-43=10)
SPNYK	NYSE Financial Stock Price Index (Avg, 2003=100)
PA15CDA5_H	Households & Nonprofit Organizations: Net Worth (NSA, Bil.\$)
WPS	Household stock market wealth, real
WPO	Household property wealth ex. stock market, real
FPX	Nominal exchange rate (G39, import/export trade weights)
FPXM	Nominal exchange rate (G39, bilateral import trade weights)
FPXR	Real exchange rate (G39, import/export trade weights)
FXUK	Foreign Exchange Rate: United Kingdom (US\$/Pound)
FXSW	Foreign Exchange Rate: Switzerland (Franc/US\$)
FXJAP	Foreign Exchange Rate: Japan (Yen/US\$)
FXCAN	Foreign Exchange Rate: Canada (C\$/US\$)
FWILL	FRB Sr Officers Survey: Banks Willingness to Lend to Consumers (%)
PTR	10-year expected inflation (Hoey/Philadelphia survey)
ZPI10	Expected cons. price infl., for RCCH and RG10E eqs. (10-yr mat., weight: 1.0)
ZPIC30	Expected cons. price infl., for RCBE and WPSN eqs. (30-yr mat., weight: 1.0)
ZPI5	Expected cons. price infl., for RG5E eq. (5-yr mat., weight: 1.0)
ZPIC58	Expected consumer price inflation (5-8 qtrs mat.)
FFED	Federal Funds [effective] Rate (% p.a.)
FBPR	Bank Prime Loan Rate (% p.a.)
FFP1	1-Month Financial Commercial Paper (% per annum)
FTB3	3-Month Treasury Bills (% p.a.)
FTB6	6-Month Treasury Bills (% p.a.)
FBDB1Y	Fama Bliss Discount Bond Yield, 1-year
FBDB2Y	Fama Bliss Discount Bond Yield, 2-year
FBDB3Y	Fama Bliss Discount Bond Yield, 3-year
FBDB4Y	Fama Bliss Discount Bond Yield, 4-year
FBDB5Y	Fama Bliss Discount Bond Yield, 5-year
TREAS1Y	Fama Bliss Treasury Yield, 1-year (fixed term index)
TREAS2Y	Fama Bliss Treasury Yield, 2-year (fixed term index)
TREAS5Y	Fama Bliss Treasury Yield, 5-year (fixed term index)
TREAS7Y	Fama Bliss Treasury Yield, 7-year (fixed term index)
TREAS10Y	Fama Bliss Treasury Yield, 10-year (fixed term index)
TREAS20Y	Fama Bliss Treasury Yield, 20-year (fixed term index)
TREAS30Y	Fama Bliss Treasury Yield, 30-year (fixed term index)
FYCCZ1E	US Treasury Yield: Continuously Compounded Zero-Coupon: 1-Yr(EOP, %)
FYCCZ2E	US Treasury Yield: Continuously Compounded Zero-Coupon: 2-Yrs(EOP, %)
FYCCZ3E	US Treasury Yield: Continuously Compounded Zero-Coupon: 3-Yrs(EOP, %)
FYCCZ4E	US Treasury Yield: Continuously Compounded Zero-Coupon: 4-Yrs(EOP, %)
FYCCZ5E	US Treasury Yield: Continuously Compounded Zero-Coupon: 5-Yrs(EOP, %)
FYCCZ6E	US Treasury Yield: Continuously Compounded Zero-Coupon: 6-Yrs(EOP, %)
FYCCZ7E	US Treasury Yield: Continuously Compounded Zero-Coupon: 7-Yrs(EOP, %)

RCAR	Commercial Real Estate: RCA-Based Top-10 MSA Retail Index DISC(NSA, Q4-00=1)
RME	Interest rate on conventional mortgages (effective ann. yield)
RG10E	10-year Treasury bond rate (effective ann. yield)
RG10P	10-year Treasury bond rate, term premium
RG5E	5-year Treasury note rate (effective ann. yield)
RG5P	5-year Treasury note rate, term premium
RG30E	30-year Treasury bond rate (effective ann. yield)
RG30P	30-year Treasury bond rate, term premium
RBBBE	S&P BBB corporate bond rate (effective ann. yield)
RBBBP	S&P BBB corporate bond rate, risk/term premium
RPD	After-tax real financial cost of capital for producers' durable equipment
RCCD	Cost of capital for consumer durables
RCCH	Cost of capital for residential investment
FK24P	Commercial Bank Interest Rates: 24-Month Personal Loans (NSA, %)
FCIR	C&I Loan Rate: All Loans, Actual (%)
FCIRS	C&I Loan Rate Spread Over Intended Fed Funds Rate: All Loans, Actual (%)
TP3M	FTB3 - FFED
TP6M	FTB6 - FFED
TP1Y	FTB1Y - FFED
TP3Y	FTB3Y - FFED
TP5Y	FTB5Y - FFED
TP7Y	FTB7Y - FFED
TP10Y	FTB10Y - FFED
TP30Y	FTB30Y - FFED
RP_BBB	RBBBE - RG10E
RP_ME	RME - RG10E
RP_CAR	RCAR - FBDB4Y
RP_24P	FK24P - FBDB2Y
FA70CNC5	Private Depository Institutions: Assets: Consumer Credit (SAAR, % of potential GDP)
FA70MOR5	Private Depository Institutions: Assets: Total Mortgages (SAAR, % of potential GDP)
FA70BLN5	Private Depository Institutions: Assets: Other Loans and Advances (SAAR, % of potential GDP)
FA76CNC0	U.S.-Chartered Depository Institutions: Assets: Consumer Credit (% of potential GDP)
FA76MOR5	U.S.-Chartered Depository Institutions: Assets: Total Mortgages (% of potential GDP)
FA76BLN5	U.S.-Chartered Dep Inst: Assets: Dep Institution Loans N.E.C.(% of potential GDP)
FA61CNC5	Finance and ABS Companies: Consumer Credit (% of potential GDP)
FA61MOR0	Finance and ABS Companies: Total Mortgages (% of potential GDP)
FA61FLB0	Finance Companies: Loans to Business (% of potential GDP)
FL14MOR5	Nonfinancial Business: Mortgages (% of potential GDP)
FL14BLN5	Nonfinancial Business: Bank Loans nec (% of potential GDP)
FL14OTL5	Nonfinancial Business: Other Loans and Advances (% of potential GDP)
FL15CNC0	Household Borrowing in Consumer Credit (% of potential GDP)
FL15OTL5	Households: Other Loans and Advances (% of potential GDP)
FL15HOM5	Household Borrowing in Home Mortgage (% of potential GDP)

FABWCA	Break-Adjusted C & I Loans in Bank Credit: All Commercial Banks (% of potential GDP)
FABWRA	Break-Adjusted Real Estate Loans in Bank Credit: All Commercial Banks (% of potential GDP)
FABWQA	Break-Adjusted Consumer Loans in Bank Credit: All Commercial Banks (% of potential GDP)
FABWOA	Break-Adjusted Other Loans & Leases in Bank Credit: All Comml Banks (% of potential GDP)
FABWCDA	Break-Adjusted C & I Loans in Bank Credit: Domestic Commercial Banks (% of potential GDP)
FABWRDA	Break-Adjusted Real Estate Loans in Bank Credit: Domestic Commercial Banks (% of potential GDP)
FABWQDA	Break-Adjusted Consumer Loans in Bank Credit: Domestic Commercial Banks (% of potential GDP)
FABWODA	Break-Adjusted Other Loans & Leases in Bank Credit: Domestic Comml Banks (% of potential GDP)
FONA	Break-Adjusted Nonrevolving Consumer Credit Outstanding (% of potential GDP)
FOTA	Break-Adjusted Consumer Credit Outstanding (% of Potential GDP)
FM1	Money Stock: M1 (SA, Bil.\$)
FM2	Money Stock: M2 (SA, Bil.\$)
NFCI	Chicago Fed National Financial Conditions Index
ANFCI	Chicago Fed Adjusted National Financial Conditions Index
LVRG	Chicago Fed National Financial Conditions Index -- Leverage
NFLVR	Chicago Fed National Financial Conditions Index -- Nonfinancial Leverage
RISK	Chicago Fed National Financial Conditions Index -- Risk
CREDIT	Chicago Fed National Financial Conditions Index -- Credit
SPVXO	Stock Market Volatility Index
EPU_HIST	Baker Bloom and Davis Historical News Based Uncertainty Index

Appendix 2

Variance computations

The dynamic model in section 4 can be cast in the format²⁷

$$\sum_{i=-\tau}^0 H_i x_{t+i} + \sum_{i=1}^{\theta} H_i E_t(x_{t+i}) = \epsilon_t \quad (6.1)$$

where τ and θ are positive integers, x_t is a vector of variables, and the H_i are conformable square coefficient matrices. The generalized saddlepath procedure of Anderson and Moore (1985) may be used to solve equation (6.1) for expectations of the future in terms of expectations of the present and the past, which may be used to substitute for the expectations in (6.1) to obtain the “observable structure” of the model,

$$\sum_{i=-\tau}^0 S_i x_{t+i} = \epsilon_t \quad (6.2)$$

Computing the k -step ahead variances for the observable variables in the models requires a few more steps. Premultiplying the observable structure by $-S_0^{-1}$, yields the *reduced form* of the structural model,

$$x_t = \sum_{i=-\tau}^{-1} B_i x_{k+i} + B_0 \epsilon_t \quad (6.3)$$

The companion system of the reduced form may be expressed in compact notation as

²⁷ This appendix borrows heavily from Fuhrer (1997).

$$y_t = Ay_{t-1} + \eta_t \quad (6.4)$$

where $y_t = [x_t, \dots, x_{t-\tau+1}]'$ and $\eta_t = [B_0\epsilon_t, 0, \dots, 0]'$. Recursively substituting equation (6.4) into itself,

$$y_{t+k} = A^k y_t + \sum_{i=1}^k A^{k-i} \eta_{t+i} \quad (6.5)$$

Because η_t is uncorrelated over time, the covariance matrix of the k -period-ahead forecasts of y_t is

$$V_t(y_{t+k}) = \sum_{i=0}^{k-1} A^i \Psi (A^i)' \quad (6.6)$$

where Ψ is the covariance matrix of η_t .