Operational Risk Quantification and Insurance

Capital Allocation for Operational Risk

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Outline

- Capital Calculation along the Loss Curve
- Hierarchy of Quantitative Models
- Small value modeling
- Extreme value modeling
- An Empirical Study: The Efficacy of Diversification
Compute capital for operational risk by introducing a convenience threshold for small value losses and a scenario threshold for extreme value losses.
LDA is based on the distribution estimation of the two random variables of severity and frequency. The estimations rely on loss and exposure data collected by risk category and by business line.

EVT provides a framework to estimate the tail of loss distributions.

LDA: Loss Distribution Approach
IMA: Internal Measurement approach
SA: Standardized approach
BIA: Basic Indicator Approach
EVT: Extreme Value Theory

- IMA corresponds to a discrete version of LDA
- SA assumes an aggregation of all risks within a business line
- BIA assumes aggregation across all risks and all business lines
Small Value Modeling 1

How to capture the unexpected loss below the threshold $T_{SV}$?

Due to high frequency nature, the aggregate distribution below $T_{SV}$ can be approximated by a normal distribution, hence the 99.9 percentile is:

$$UL_{T_{SV}} = 3.1 \times \sigma_S$$

$UL_{T_{SV}}$ Unexpected loss below $T_{SV}$

$\sigma_S$ Standard deviation of aggregate loss

Standard deviation of aggregate loss can be expressed as:

$$\sigma_S = \sqrt{E[N] \text{VAR}(X) + \text{VAR}(N)E[X]^2}$$
Small Value Modeling 2

What is the order of magnitude of $UL_{T_{SV}}$?

Assume all loss be equal to threshold: $X = T_{SV}$

$UL_{T_{SV}} = 3.1 \times \sigma_N \times T_{SV}$

Assume a Poisson frequency and

$T_{SV} = 10'000$
$E[N] = 10'000$

$UL_{T_{SV}} = 3'100'000$
For frequency modeling of small losses it is important to distinguish between a Poisson or a Negative Binomial distribution.

Poisson makes the assumption that losses incur independently. In particular the variance of the frequency is equal to its mean.

In the case of Negative Binomial different events may depend on one and the same cause. At the same time the variance of frequency is assumed to be higher than its mean.

Definition:

\[ q = \frac{\text{Var}(N)}{\mathbb{E}[N]} \]

**Abrupt changes in the frequency need to be considered!**

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Year</th>
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<tr>
<td>90</td>
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<tr>
<td>98</td>
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<td>110</td>
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<td>85</td>
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<td>105</td>
<td></td>
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<td>200</td>
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Assume \( X = T_{SV} = 5'000 \)

\[ \mathbb{E}[N] = 98 \]
\[ q = 1.1 \]

\[ UL_{T_{SV}} = 160 '000 \]

Results in an additional unexpected loss of:

\[ (200-98) \times 5'000 = 510'000 \]

\[ \mathbb{E}[N] = 115 \]
\[ q = 16 \]

\[ UL_{T_{SV}} = 664 '000 \]
Small Value Modeling 4

**How to account for small value events?**

- If frequency is stable apply Poisson distribution, otherwise
- assume frequency to be Negative Binomial, or
- if the increase in frequency can be explained by one and the same cause, an aggregation of these events to a single one may be considered

As a rule of thumb the $q$ factor can be determined by assuming an appropriate multiple of the expected frequency (2-3 times) and subsequently computing the implied variance.

- Poisson with $q \approx 1$
- Plus a high severity low and frequency scenario

\[ q > 1 \]
Extreme Value Modeling 1

The general conditions refer to the maximum domain of attraction (MDA) property.

MDA property requires that the distribution of the normalized maxima, for any finite set of samples, converges as the sample size increases.

This property is satisfied for a wide class of distributions applied in insurance.

**Pickands-Balkema-de Hann Theorem**

Under some general conditions the limiting distribution of the excesses over a high threshold \( u \),

\[
F_u(x) = P[X - u \leq x|X > u] = \frac{F(x + u) - F(u)}{1 - F(u)},
\]

is either the Second Pareto or the exponential distribution:

\[
F_u(x) \xrightarrow{u \to \infty} GP(x) = \begin{cases} 
SP(x) = 1 - \left(\frac{\omega}{\omega + x}\right)^\rho \\
E(x) = 1 - \exp(-\rho x)
\end{cases}
\]

Notation: \( u = T_{EV} \)
Tail Distribution

The tail of the original distribution above threshold is obtained by fitting either a Second Pareto or an Exponential distribution to the excesses and applying:

$$F(x) = F(u) + (1 - F(u)) \cdot GP(x-u) \quad \text{for} \quad x > u$$
How to obtain the tail distribution?

Distinguish two cases:
• internal data provides sufficient number of excess losses
• external data is required to gap lack of excess losses

In practice:
• excess losses of different business lines and risk categories may be combined to obtain a sufficient number of excess events
  one tail model for the organization
How to obtain the tail frequency?

- In case of sufficient internal data, apply the historical excess frequency.
- In case of external loss data, either:
  - take the extrapolated excess frequency of internal models
  $$N_{EV} = N \times (1 - F(T_{EV}))$$
  - or scale the excess frequency of external data by utilizing exposure information
  $$N_{T_{EV}} = \frac{\sum N_i}{\sum E_{i, \text{other banks}}} \times E_{\text{Bank}}$$
Aggregation of the internal LDA models and the extreme value model results in the overall loss model.

The overall model can be used to compute relevant quantities such as unexpected loss level.

Note that as an alternative we can first compound each severity with the respective frequency distribution and subsequently perform an aggregation of the resulting distributions. However, the latter approach is, however, computationally more costly.

### How to bring things together?

Truncated severity distribution and the frequency distribution resulting from internal LDA: \((N, F)\)

Excess severity of EVT and excess frequency: \(\left(N_{TEV}, F_{TEV}\right)\)

\[
\begin{align*}
N_{total} &= N + N_{TEV}, \\
F_{total} &= \frac{N}{N_{total}} F + \frac{N_{TEV}}{N_{total}} F_{TEV}
\end{align*}
\]
An Empirical Study: The Efficacy of Diversification

Data sources

• 84 publicly reported losses in excess of USD 50mn

• OECD Statistics: Bank Profitability 2000
  – Aggregate gross income
  – Aggregate Tier1 and Tier2

• Global Researcher Worldscope data base
  – Balance sheet positions of individual institutions
An Empirical Study: Efficacy of Diversification 2

The linear regression analysis is performed for the aggregate quantities. It suggests that the dependency may also be valid if single institutions are considered.

Similar analysis, however, needs to be conducted to validate the assumption of correlation at the level of single institutions.

Following plot depicts the frequency of losses in excess of USD 50 millions versus gross income of G7 commercial banks.

\[ Freq = 0.07 \times GI - 23.5 \]

\[ R^2 = 64\% \]
An Empirical Study: Efficacy of Diversification 3

By applying EVT distributions we obtain:

\[ SP(x) = 1 - \left( \frac{194377}{194377 + x} \right)^{1.507} \]

As an example we compute the capital at 99 percentile for year 1998 assuming the following two scenarios:

- \[ F(T_{EV}) = 99.5 \% \Rightarrow UL_{div} = 12'110 \text{ mn} \]
- \[ F(T_{EV}) = 99 \% \Rightarrow UL_{div} = 17'040 \text{ mn} \]

These figures already take into account the effect of diversification!
An Empirical Study: Efficacy of Diversification 4

To obtain the undiversified capital, we need to compute the gross-income-weighted number of institutions.

The number of commercial banks in G7 for year 1998 was 9'862.

From the distribution of gross income we obtain approximately 450 institutions as the gross-income-weighted number.
The undiversified capital is computed by means of

\[ UL = \sqrt{450} \times UL_{div} \]

For comparison consider the ratio of capital to current Tier1+Tier2 level

<table>
<thead>
<tr>
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<th>Diversified</th>
<th>Undiversified</th>
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<tbody>
<tr>
<td>( F(T_{EV}) = 99% )</td>
<td>1.3%</td>
<td>27%</td>
</tr>
<tr>
<td>( F(T_{EV}) = 99.5% )</td>
<td>0.9%</td>
<td>20%</td>
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Previous analysis suggests a risk sensitive calibration of BIA.
Fig. 1: Second Pareto fit of excess losses

Fig. 2: Original distribution recovered by assuming that 99.5% of losses are below the excess threshold

Fig. 3: Aggregation of distribution in Fig.2 with the estimated frequency 11.7/(1-0.995)
An Empirical Study: Efficacy of Diversification

Insurance as an intermediate solution between the diversified and undiversified worlds achieves the diversification benefit in an efficient way.

An analogous example, is provided by insurance linked securitization.

In contrast to risk transfer to an insurer, securitized risks are transferred to capital markets. In case of a securitization the amount of funds provided by investors are equal to capacity guaranteed by the transaction.

Whereas, the same risk, if transferred to an insurer, would require less “funds.”