Comments on

The Role of Inventories and Speculative Trading in the Global Market for Crude Oil

by Lutz Kilian and Dan Murphy

SVAR:

$$B_0 Y_t = B(\mathrm{L}) Y_{t-1} + \varepsilon_t$$

Identification Issues:

1. What are the shocks that KM are trying to estimate?

2. What restrictions do KM impose on the SVAR to estimate these shocks?

$$B_0 Y_t = B(\mathrm{L}) Y_{t-1} + \varepsilon_t$$

Impact Effects (weak)

	Unobserved Shocks			
Observables	$\varepsilon_{Flow\ Supply}$	$\varepsilon_{Flow\ Demand}$	$\varepsilon_{SpeculativeDemand}$	ε_{Other}
Oil Production	↓	↑	↑	
Real Activity	↓	↑	↓	
Price of Oil	↑	↑	↑	
Inventories			↑	

ε : VAR Unanticipated Shocks

$\varepsilon_{Speculative\ Demand}$ captures shocks to Flow Supply and Flow Demand that are anticipated using wider information set. ("Omitted Variable Bias").

Identifying SVARs using Sign Restrictions, etc.

SVAR: $B_0 Y_t = B(\mathrm{L})Y_{t-1} + \varepsilon_t$

VAR: $Y_t = A(\mathrm{L})Y_{t-1} + e_t$ where $A(L) = B_0^{-1}B(L)$ and $e_t = B_0^{-1}\varepsilon_t$

Dynamic Simultaneous Equation Parameterization of Bivariate SVAR:

$$Y_{1t} = -b_{0,12}Y_{2t} + \text{lags} + \varepsilon_{1t}$$

$$Y_{2t} = -b_{0,21}Y_{1t} + \text{lags} + \varepsilon_{2t}$$

Restrictions: $\mathrm{E}(\varepsilon_{1t}\varepsilon_{2t}) = 0$ and B_0 has 1's on diagonal.

1 additional restriction needed for identification

$$Y_{1t} = -b_{0,12}Y_{2t} + \text{lags} + \varepsilon_{1t}$$

$$Y_{2t} = -b_{0,21}Y_{1t} + \text{lags} + \varepsilon_{2t}$$

Set Identification:

Suppose $b_{0,12} = 0.0$ … compute implied SVAR
Suppose $b_{0,12} = 1.0$ … compute implied SVAR
Suppose $b_{0,12} = 3.14$ … compute implied SVAR
…

Suppose $Y_t = AY_{t-1} + e_t$, with A and Σ_e known.

Let γ denote a set of IRFs or other parameters of interest. Then $\gamma = \gamma(A, \Sigma_e, b_{0,12})$,

Identified Set is: $\Gamma = \{\gamma \mid \gamma = \gamma(A, \Sigma_e, b_{0,12}) \text{ and } -\infty \leq b_{0,12} \leq \infty\}$

Impulse Responses: $Y_t = AY_{t-1} + e_t$ where $e_t = B_0^{-1}\varepsilon_t$

Suppose $\frac{\partial Y_{i,t+k}}{\partial \varepsilon_{jt}}$ is restricted, $\frac{\partial Y_{i,t+k}}{\partial \varepsilon_{jt}} = \left[A^k B_0^{-1} \right]_{ij}$

With A known, this imposes restrictions on B_0.

Set Identification:

Suppose $b_{0,12} = 0.0$ … compute implied SVAR (violates sign restriction)

Suppose $b_{0,12} = 1.0$ … compute implied SVAR (OK sign restriction)

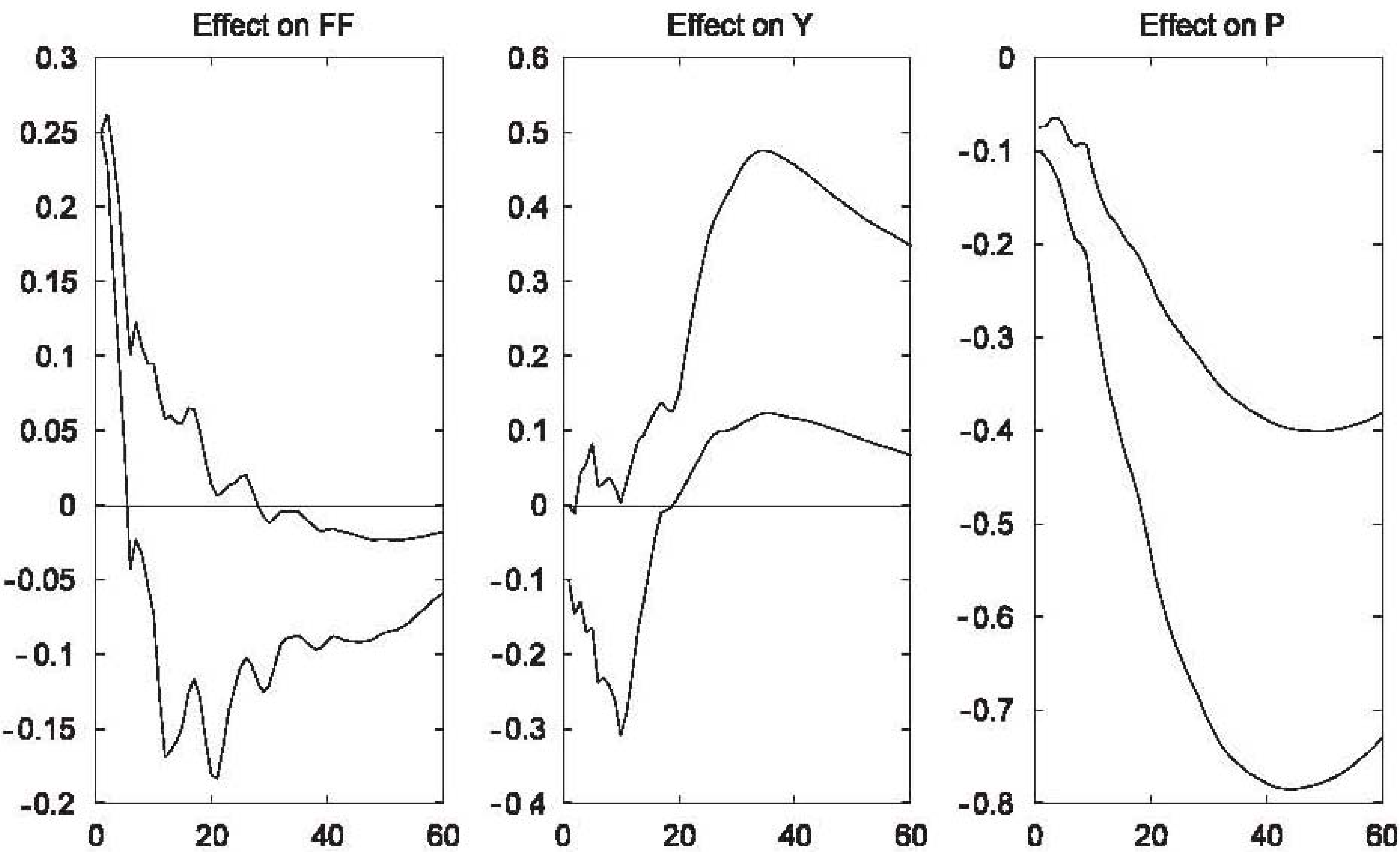
Suppose $b_{0,12} = 3.14$ … compute implied SVAR (OK sign restriction)

etc.

Sign Restrions imply $b_{0,12} \in B_R$

Identified Set: $\Gamma = \{\gamma \mid \gamma = \gamma(A, \Sigma_e, b_{0,12}) \text{ and } b_{0,12} \in B_R\}$

Estimates of identified sets (Faust, Swanson, Wright (2003), Fig 3)



$Y_t = AY_{t-1} + e_t$

Dynamic Simulatenous Parameterization:

$e_t = B_0^{-1}\varepsilon_t$, B_0 has ones on diagonal, Σ_ε is diagonal

Alternative Parameterization: $e_t = C\varepsilon_t$, $\Sigma_\varepsilon = \mathrm{I}$

Identification using alternative paramterization:

$\Sigma_e = C\Sigma_\varepsilon C' = CC' = CR(CR)'$ for any R that satisfies $RR' = \mathrm{I}$

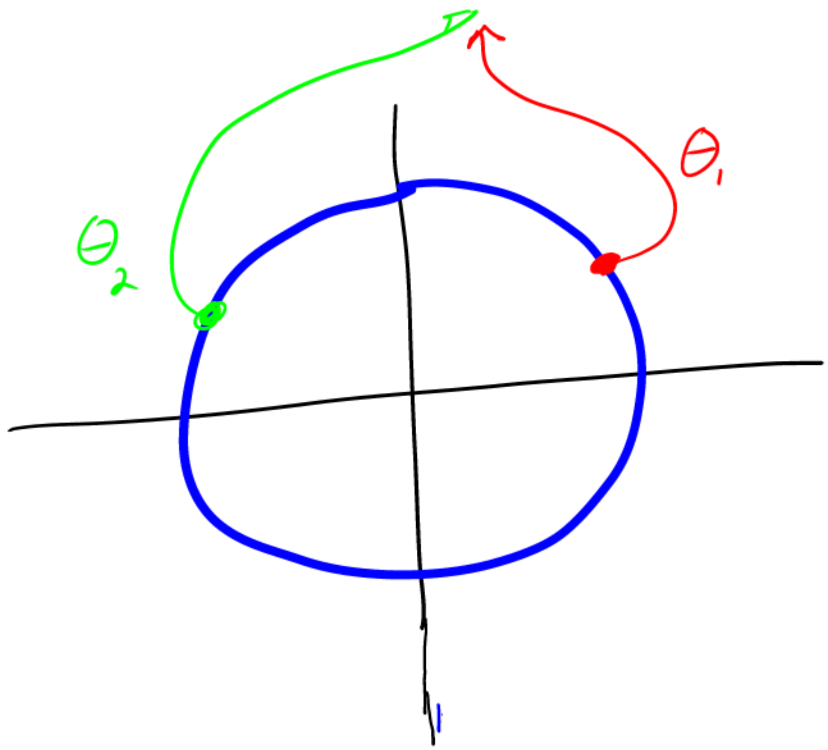
Thus $C = \Sigma_e^{1/2}R$, where $\Sigma_e^{1/2}$ is the Cholesky factor of Σ_e and R is an orthonormal matrix. Bivariate model

$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, so θ indexes the set of identified models.

Mechanics: Suppose $Y_t = AY_{t-1} + e_t$, with A and Σ_e known.

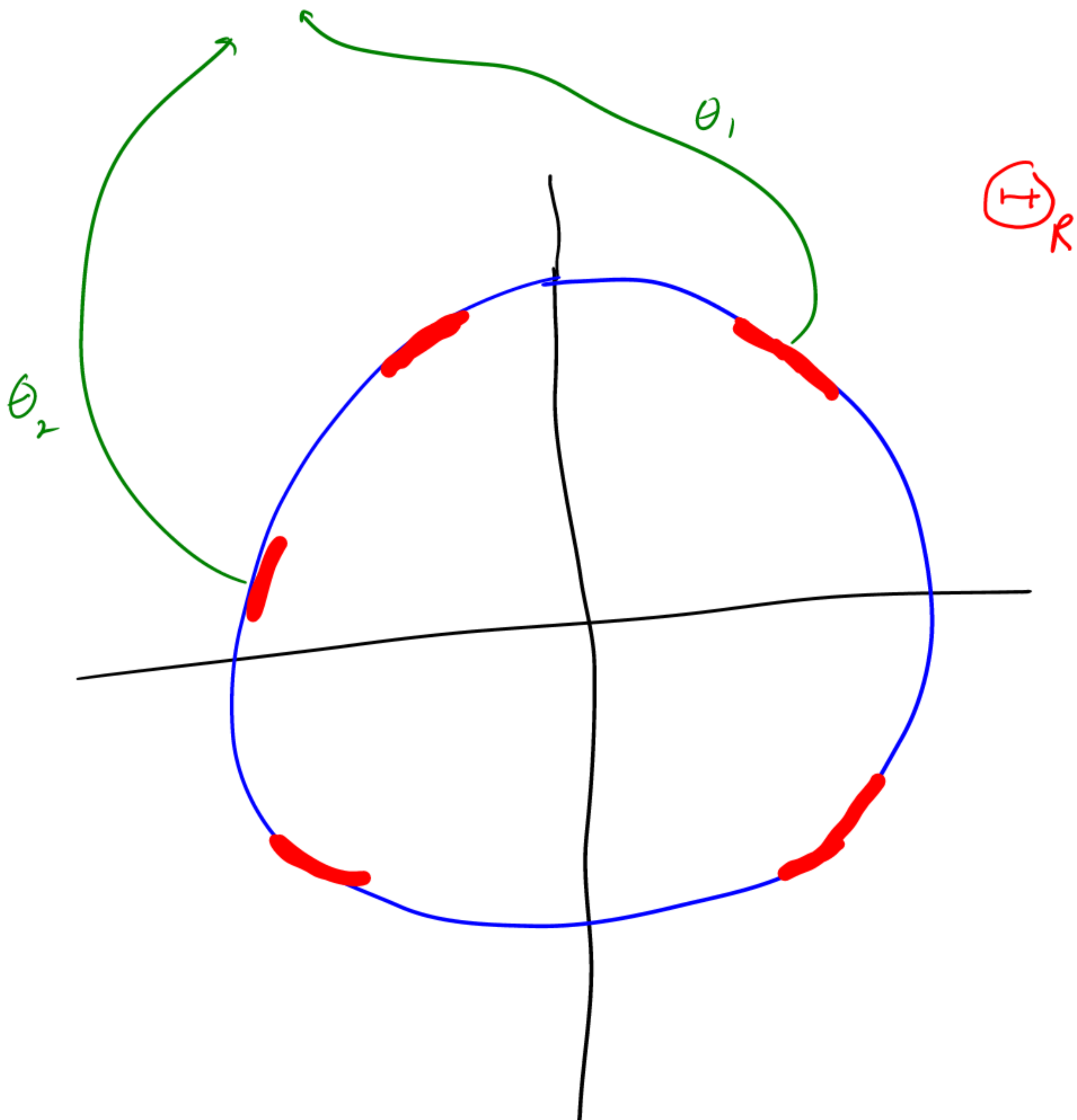
Let γ denote a set of IRFs or other parameters of interest. Then $\gamma = \gamma(A, \Sigma_e, \theta)$,

Identified Set is: $\Gamma = \{\gamma \mid \gamma = \gamma(A, \Sigma_e, \theta) \text{ and } 0 \leq \theta \leq 2\pi)$



Sign Restricted Identified Set: $\theta \in \Theta_R$, so that

$\Gamma = \{\gamma \mid \gamma = \gamma(A, \Sigma_e, \theta)$ and $\theta \in \Theta_R)$.



Uncertainty and Identified Set:

All points in Γ are consistent with data. Data can't be used to say anything about which points are more "likely".

Bayesian: Prior knowledge about the parameter that indexes the identified set ($b_{0,12}$ or θ). Use this prior to assign weights/probabilities to points in identified set. Compute "Averages", "Credible Sets", and so forth.

2 Key things:

(1) Results are based on the prior, not the data. Thus results are only as good as the prior.

(2) Priors can be subtle in nonlinear models like these.

How are priors on $b_{0,12}$ and θ related?

Bivariate Model: $\Sigma_e = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$

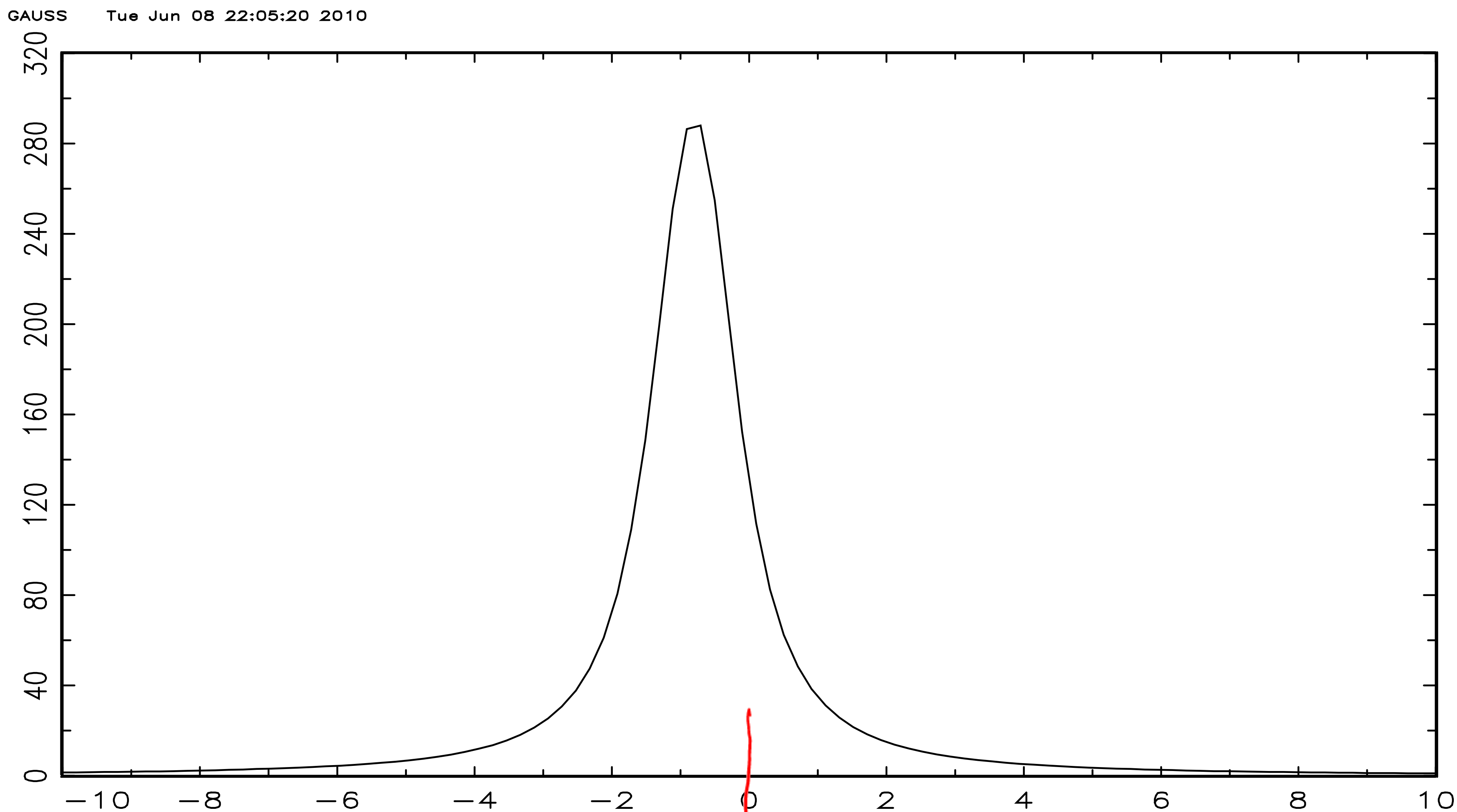
Simultaneous equation model: $Y_{1t} = -b_{0,12}Y_{2t} + \text{lags} + \varepsilon_{1t}$

Rotation: $\Sigma_e = \Sigma_e^{1/2}R(\theta)R(\theta)'\Sigma_e^{1/2\,\prime}.$

$b_{0,12} = b(\theta)$, where b is a nonlinear function

Suppose Prior on θ is uniform. What is implied prior on $b_{0,12}$?

Prior: θ uniform on 0 to π … Implied prior for $b_{0,12}$ … $\Sigma_e = \begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix}$



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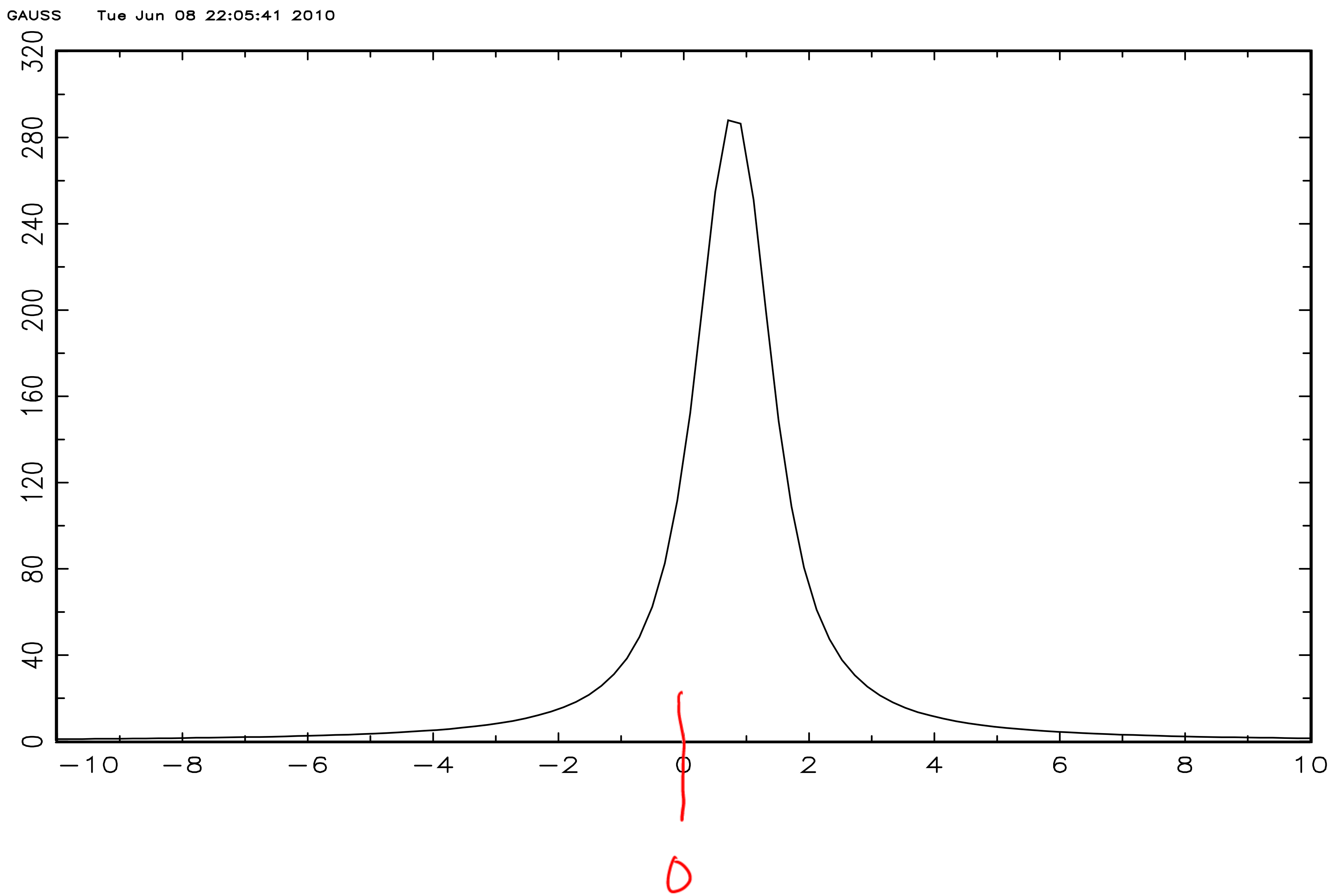
Prior: θ uniform on 0 to π … Implied prior for $b_{0,12}$ … $\Sigma_e = \begin{bmatrix} 1 & -0.9 \\ -0.9 & 1 \end{bmatrix}$



Prior on θ is flat and does not depend on Σ_e.

Implied Prior on $b_{0,12}$ is not flat, not symmetric, and depends on Σ_e.

VAR: $Y_t = A(\mathrm{L})Y_{t-1} + e_t$

What about sampling uncertainty about $A(\mathrm{L})$ and Σ_e.

Identified set: $\Gamma = \{\gamma \mid \gamma = \gamma(A, \Sigma_e, b_{0,12}) \text{ and } b_{0,12} \in B_R\}$

Frequentist Confidence Set over identified sets:

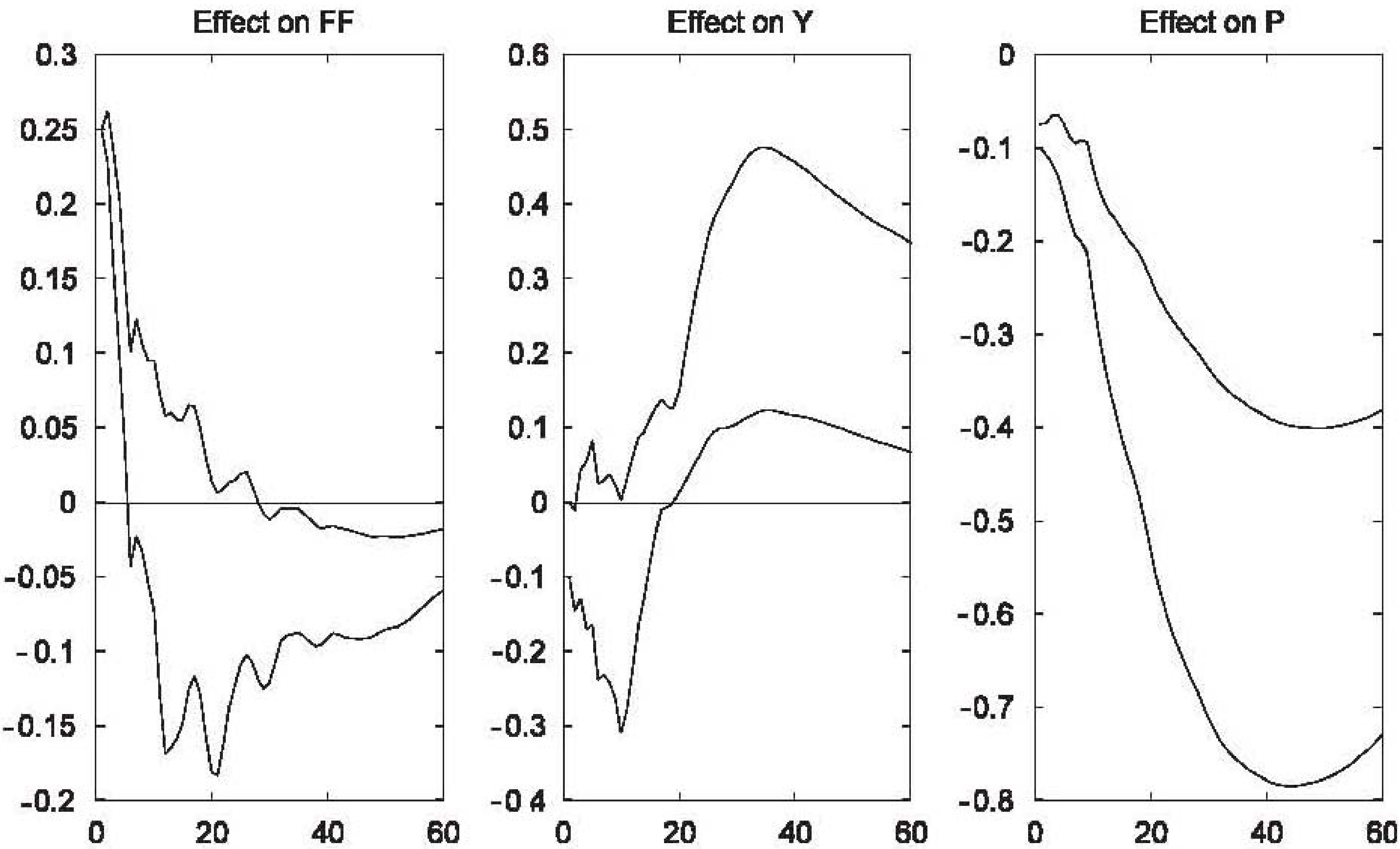
Let $\Xi\,(Y)$ denote a 95% set for A, Σ_e … $P\left[(A,\Sigma_e) \in \Xi\right] = 0.95$

Confidence set : $\Gamma = \{\gamma \mid \gamma = \gamma(A, \Sigma_e, b_{0,12}),\ (A,\Sigma_e) \in \Xi, \text{ and } b_{0,12} \in B_R\}$

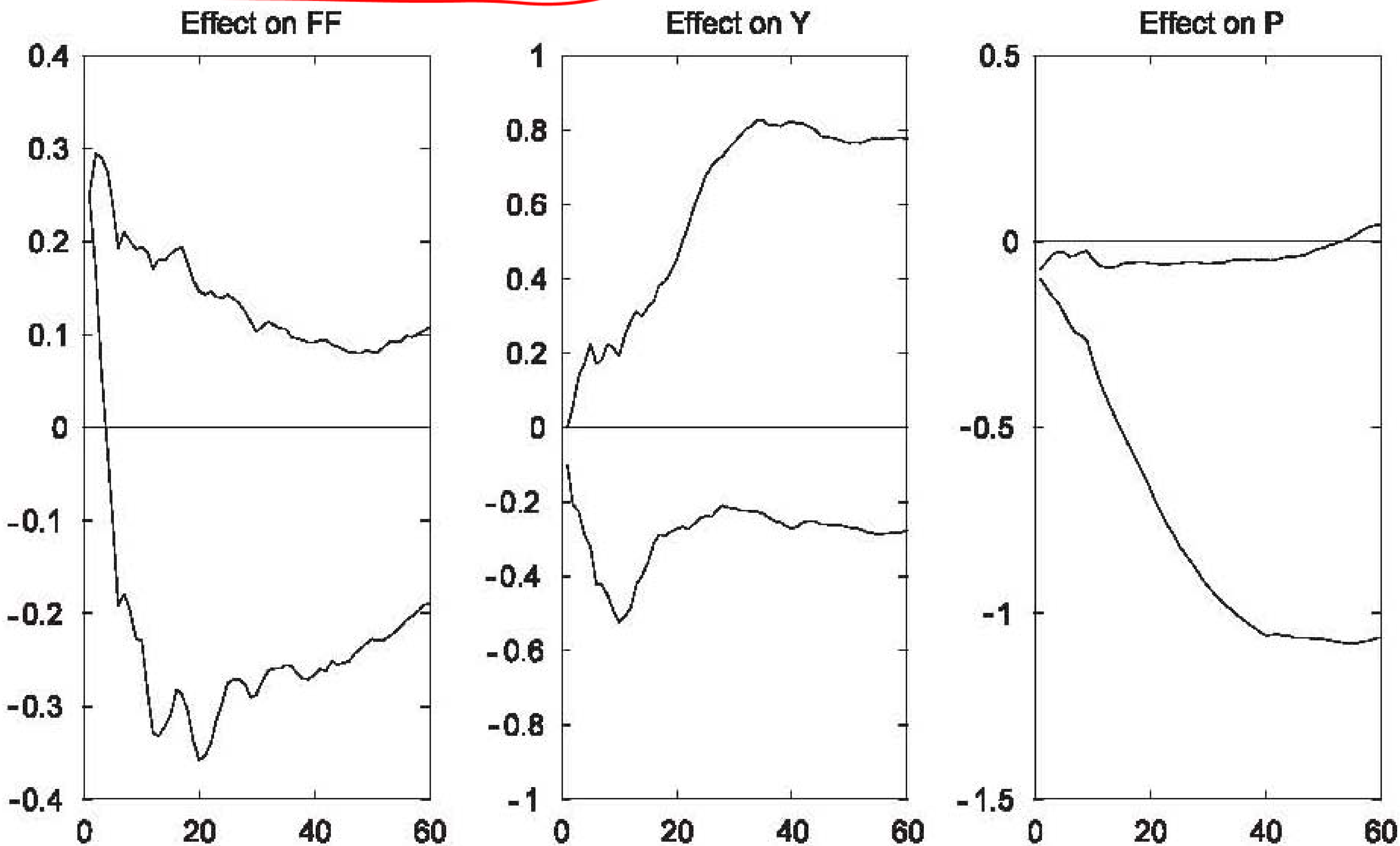
Moon, Schorfheide, Granziera, Lee (2009)

Faust, Swanson, Wright (2003)

Estimates of identified sets (Faust, Swanson, Wright (2003), Fig 3)



Confidence sets for identified sets (Faust, Swanson, Wright (2003) , Fig 4)



Lessons:

- Reporting Results On Identified Sets is nonstandard.
- There aren't frequentist "point estimates" to report
- Bayesian results (point estimates as posterior means, credible sets) depend critically on priors. Priors are subtle.

Returning to Kilian and Murphy …

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