An Evaluation of the Federal Reserve Estimates of the Natural Rate of Unemployment in Real Time

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Abstract: We derive an estimate of the Federal Reserve's assessment of the natural rate of unemployment in real-time from the Greenbook forecast of inflation. The estimated natural rate starts to rise noticeably in the mid-1970s. It stays relatively high in the 1980s, and then declines pronouncedly in the second half of the nineties. We compare the Greenbook estimates with the estimates obtained in real-time from simple relationships that extract information about the natural rate of unemployment from the dynamics of inflation, aggregate demand, and the functioning of the labor market. When differences between these measures and the Greenbook measure arise, the improvement in the Greenbook inflation forecast that would have been achieved by using a different estimate of the natural rate of unemployment is typically small.

JEL Classifications: E37, E47, E5

1. *Introduction*

The assessment of an economy's degree of resource utilization is an important input to the conduct of monetary policy. This is the case not just when the monetary policy authority has, in addition to price stability, a full-employment mandate, but also when the mandate is specified only in terms of an inflation goal. As long as there is a link between the degree of resource utilization and inflation, inference about the amount of slack in the economy is in fact a relevant component of the inflation outlook, and as such it informs the conduct of monetary policy.

It is not always straightforward to evaluate how far the economy is from full employment, or from the natural rate of unemployment. This is especially true when the economy moves markedly away, and for an extended period of time, from a pre-existent notion of equilibrium. The high and persistent rates of unemployment in the most recent recession and recovery episodes have generated much debate on whether the natural rate of unemployment has changed, too. Inference about the degree of resource utilization has been problematic also in the past, and missteps in the conduct of monetary policy have often been attributed to an incorrect assessment of the distance of the economy from full employment.

The difficulties in assessing the degree of slack in an economy in real time have been documented extensively when the slack is measured by the output gap. Analysis of the Federal Reserve's staff real-time assessment of the output gap in the 1980s and 1990s also points to the unreliability of the staff estimates.[1] Less work, however, has been devoted to estimating economic slack in real time from a labor market perspective. Okun's law relates the output gap to the unemployment rate gap, and thus the uncertainty about the real-time output gap has to translate, to some extent, into uncertainty about the real-time unemployment rate gap. Moreover, there is a large literature illustrating that estimates of the unemployment rate gap have a considerable degree of uncertainty, even when these estimates benefit from data not available in real time.[2] Nevertheless, focusing on the unemployment rate gap can have advantages. Contrary to GDP, the unemployment rate does not get revised. There is also evidence indicating that the unemployment rate has predictive power for future revisions to GDP relative to the real-time GDP reading.[3] Even more importantly, real-time inference about the unemployment rate gap can be drawn not just from the typical aggregate macro relationships such as the Phillips curve, but also from the functioning of the labor market.

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[1] See, among others, Orphanides and Van Norden (2002) and Orphanides (1998).

[2] See, among others, Staiger, Stock, and Watson (1997).

[3] See Aruoba (2008).

A notion of the degree of resource utilization based on the labor market is also central to the FOMC's current conduct of monetary policy. Recent FOMC's policy statements have squarely focused, in addition to inflation developments, on progress in the labor market as a guide for policy. Indeed, current guidance for the timing of the liftoff of the federal funds rate from the zero lower bound is based on the economy reaching a specific value for the unemployment rate in a context of stable inflation. This kind of policy guidance relies on some notion of how the targeted value of the unemployment rate for the liftoff date relates to the natural rate of unemployment. Too, the recent debate about the amount of slack in the economy has been heavily influenced by observations pertaining to the functioning of the labor market.

For all of these reasons, in this paper we revisit the issue of estimating activity slack in real-time by focusing on real-time measures of the natural rate of unemployment. In particular, the paper provides an assessment of the Federal Reserve's staff estimate of the natural rate of unemployment. This estimate is readily available in the most recent years, when the Greenbook explicitly stated the staff's assumptions about the natural rate of unemployment. It is not, however, in earlier periods. It is then necessary to infer the staff's view about the natural rate of unemployment, and we do so by backing out an estimate of Greenbook's natural rate of unemployment from the Greenbook inflation forecast. We then compare the Greenbook estimate with estimates obtained in real-time from simple benchmark relationships. These relationships derive a measure of the natural rate of unemployment by estimating jointly a Phillips curve and an aggregate demand equation, and a Phillips curve and a Beveridge curve relationship. We find that the Greenbook estimates of the natural rate of unemployment are broadly consistent with these real-time estimates. There is little suggesting that the Greenbook estimates of the natural rate of unemployment have been systematically lagging behind. When differences between the simple benchmark real-time and the Greenbook estimates arise, the improvement to the Greenbook inflation forecast that would have been achieved by using a different estimate is typically small. Too, this result depends on the sample period being considered.

The rest of the paper proceeds as follows. In section 2 we illustrate our method for backing out estimates of the natural rate of unemployment from the Greenbook inflation forecast and discuss these estimates. In section 3 we consider a real-time exercise for estimating the natural rate of unemployment from simple relationships. Section 4 compares the estimates obtained in section 3 with the Greenbook estimates. Performance of the different estimates is assessed in terms of potential improvement to the Greenbook inflation forecast. Section 5 offers some concluding remarks, pertaining in particular to how

our assessment of the Greenbook estimates of the natural rate of unemployment would change if, instead of comparing the Greenbook estimates to real-time estimates, the comparison were made with ex-post estimates of the natural rate of unemployment.

2. *Greenbook Estimates of the Natural Rate of Unemployment*

In this section we extract real-time estimates of the natural rate of unemployment from the Federal Reserve economic projections reported in the Greenbook. The Greenbook forecast of the U.S. economy is produced by the research staff of the Federal Reserve Board before each FOMC meeting to support FOMC members in their policy deliberations. While an assessment of the size of the activity gap, be it in the form of an unemployment rate or in the form of an output gap, is a crucial element in the conduct of monetary policy, the Greenbook has been reporting a real-time assessment of the natural rate of unemployment in a consistent manner only since the 1990s. For earlier periods, when such an assessment was not readily available, it is necessary to draw inference about the Board staff's views of the real-time activity gap. The inference exercise retains some value even for the more recent period when the Board staff started to provide real-time estimates of the natural rate of unemployment. This has to do with the fact that the Greenbook forecast is in part judgmental. As a result, the inference exercise provides an assessment of the extent to which the Greenbook forecast conforms with the notion of the natural rate of unemployment.

We infer the staff's real-time assessment of the natural rate of unemployment from the Greenbook forecast of inflation. The basic premise of this exercise is that the activity gap plays an economically relevant role in driving inflation, and that this is reflected in how the Board staff approaches the inflation forecast. In essence, we posit that the Greenbook inflation forecast can be described by a Phillips curve relationship, where the aggregate demand measure is defined in terms of an unemployment rate gap. Such a relationship is estimated on the Greenbook forecast of inflation, using as explanatory variables information available in real time. The equilibrium rate of unemployment is then backed out from the estimated relationship in the same way as it is when the relationship is estimated on actual data.

This is not the first study that tries to infer the Greenbook's views about the natural rate of unemployment from a relationship that links real activity to inflation. In particular, Romer and Romer (2002) perform the same type of exercise using a simple back-of-the-envelope calculation that links changes in inflation to the deviation of the unemployment rate from its natural rate. Such an exercise

results in noisy estimates of the natural rate of unemployment.[4] Our approach imposes more structure, and arguably produces a somewhat clearer picture of the Federal Reserve's assessment of the natural rate of unemployment in real time.

We estimate the following Phillips curve relationship on Greenbook forecasts of inflation:

$$(1) \qquad \pi_{t+4}^{4,GB} = \left(\alpha \bar{\pi}_t^{RT} + (1-\alpha)\pi_t^{4,RT} \right) - \beta \left(u_t^{RT} - \bar{u}_t \right) + x_t^{RT}\delta + \varepsilon_t,$$

where $\pi_{t+4,t}^{GB}$ denotes the Greenbook's time t forecast of inflation over the next four quarters (with four-quarter inflation at any date t defined as $\pi_t^4 \equiv 0.25 * \sum_{i=0}^{3} \pi_{t-i}$). The independent variables in (1) are denoted with a superscript "*RT*" to indicate that they are observed in real-time and are thus in the Greenbook's information set when forecasting inflation. The first term in brackets on the right-hand side of (1) captures inflation expectations in the Phillips curve. We assume that these expectations are given by a weighted average between a measure of long-run inflation expectations, $\bar{\pi}_t^{RT}$, and the average rate of inflation prevailing in the most recent 4 quarters, $\pi_t^{4,RT}$. The inclusion of long-run inflation expectations is meant to capture the FOMC's inflation goal. The dependence of inflation expectations in the Phillips curve relationship (1) on a long-run expected measure of inflation does not necessarily imply an exploitable tradeoff between inflation and unemployment. Long-run inflation expectations could in fact respond quickly to certain inflation developments and/or policy actions. In this setup, the accelerationist view of inflation is nested as a limiting case when $\alpha = 0$.

The second term in the equation is the unemployment rate gap, where u_t^{RT} denotes the unemployment rate and $\bar{u}_t$ is the unobserved natural rate of unemployment that we are interested in estimating. Finally, x_t^{RT} is a vector of supply shocks. The relationship in (1) can be interpreted as featuring a time-varying intercept, $\beta\bar{u}_t$, which captures fluctuations in the natural rate of unemployment. In order to back out an estimate of $\bar{u}_t$ from (1), we assume that $\bar{u}_t$ evolves over time as a random walk

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[4] See their Chart 2 on page 47.

(2) $\bar{u}_t = \bar{u}_{t-1} + v_{u,t}$

where the shock $v_{u,t}$ is uncorrelated with the shock ε_t.

In the context of the Phillips curve relationship (1), the natural rate of unemployment can be thought of as the rate of unemployment that in the medium term stabilizes inflation at the level consistent with the perceived inflation goal. While we are backing out $\bar{u}_t$ from the Greenbook by assuming that the Greenbook outlook for inflation is based on a Phillips curve relationship such as the one described in (1), we are not implying that the Greenbook assessment of the natural rate of unemployment is informed exclusively by the relationship between inflation and real activity. Greenbook estimates of the natural rate of unemployment are likely based on many different sources of information, such as micro-level information pertaining to the functioning of the labor market. However, as long as the Greenbook's estimate of the natural rate of unemployment is a factor affecting the inflation forecast, and the additional sources of information are largely uncorrelated with the other right-hand side variables, it is sufficient to consider the inflation forecast to infer the Greenbook's assessment of the natural rate.

Data

Up to the end of 1985, Greenbook inflation forecasts refer to the GNP implicit price deflator. Starting in 1986, the inflation forecast refers to the core CPI. In the analysis, we use all of the Greenbooks which feature the four quarters of inflation forecasts necessary to construct $\pi_{t+4}^{4,GB}$. Greenbook forecasts become publicly available with a five-year lag since publication, and as a result our analysis covers the period 1970 to 2007. Real-time information on the most recent four quarters of inflation, $\pi_t^{4,RT}$, is as reported in each Greenbook, and so is the value for the unemployment rate u_t^{RT}. Supply shocks are captured by the change in real oil prices and by the change in real food prices. For $\bar{\pi}_t^{RT}$, ten-year measures of inflation expectations are available only from the end of 1979. We use the Blue-Chip 10-year CPI inflation expectations from 1979:Q4 to 1991:Q2. Because these expectations were surveyed only twice a year, we interpolate for the missing quarters. From 1991:Q4 onward, we use the Survey of Professional Forecasters 10-year CPI inflation expectations.[5] For the earlier period, we simulate real-time long-run inflation expectations according to the following relationship:

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[5] The historical data for the 10-year inflation expectations from the Blue Chip Economic Indicators and from the Survey of Professional Forecasters are maintained by the Federal Reserve Bank of Philadelphia, at http://www.philadelphiafed.org/research-and-data/real-time-center/.

(3) $\bar{\pi}_t^{RT} = 0.965 * \bar{\pi}_{t-1}^{RT} + 0.0355 * \pi_{t,t-3}^{4,RT}$.

The starting point for the simulation is 1955:Q4, with $\bar{\pi}_{1955:Q4}^{RT}$ set at 1.7 percent. Given this starting

point, simulated values are generated up to 1979:Q4, when $\bar{\pi}_{1979:Q4}^{RT}$ reaches a reading that is very close

to the first available data point for 10-year inflation expectations. The simple relationship assumed in (3)

mimics well the Federal Reserve Board's FRB/US model estimate of 10-year inflation expectations pre-

1980.[6]

Estimation

Equations (1) and (2) are in state-space form, where (1) is the measurement equation and (2) is the state

equation. Estimates of the relevant parameters can be obtained via maximum likelihood by

implementing the Kalman filter. While we allow explicitly for time variation in the estimated natural rate

of unemployment, the other coefficients in (1) are fixed over time. To capture some low-frequency

changes in these other coefficients, we estimate (1) and (2) over four different subsamples. The first

subsample covers the 1970s. Since 1985 is the last year for which Greenbook inflation forecasts refer to

the GNP deflator, the next subsample considers the years from 1980 to 1985. We then split the

remaining years into the period 1986 to 1996, and the period 1997 to 2007. We break the sample at the

end of 1996 to allow for the possibility that around that time the slope of the Phillips curve became

flatter.

At the estimation stage of a model such as the one in (1) and (2), relevant issues arise when assessing

the standard error of the innovation to the time-varying component in (2), $\sigma(v_u)$. These issues, and

ways of addressing potential biases in in the estimation of $\sigma(v_u)$, have been widely discussed in the

literature.[7] Here, rather than estimating $\sigma(v_u)$, we calibrate the value to equal to 0.07 in the first three

subsamples that we consider and 0.045 in the last subsample. The lower standard deviation for the

latest subsample reflects the fact that during that period fluctuations in the unemployment rate have

been relatively modest. Studies that back out the natural rate of unemployment for the United States

from *actual* inflation data using a similar setup as the one considered in (1) and (2) rely on larger values

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[6] The 10-year inflation expectations series in FRB/US has the mnemonic 'PTR.' Estimates of inflation expectations
pre-1980 are obtained in FRB/US by backcasting a model of survey expectations fitted over the period 1981 to
2006. The relationship in (3) has been estimated over the period 1968:Q1 to 1979:Q4 using the 'PTR' series as the
dependent variable. The R^2 from the estimated relationship is 0.97.

[7] See, for example, Stock and Watson (1998).

for the standard deviation of v_u than the ones we have calibrated.[8] However, Greenbook forecast are typically available 8 times a year.[9] This higher frequency of the Greenbook forecasts relative to the quarterly frequency used in these other studies on actual inflation justifies a smaller calibrated $\sigma(v_u)$. The disturbance ε_t in (1) is modeled as a first-order moving average process to account for the serial correlation generated by the overlap in the one-year-ahead inflation forecasts.

Results

Figures 1 to 4 depict the one-sided estimates of $\bar{u}_t$ against the unemployment rate for the four different subsamples we consider.[10] For the 1970s (shown in figure 1), the estimated natural rate of unemployment increases from about 4½ percent at the beginning of the sample to 7 percent in 1979. The standard error of the final observation in the sample for $\bar{u}_t$, which provides some indication of the imprecision of the one-sided estimates, is 0.4. A nontrivial portion of the increase in the estimated natural rate occurs in the mid-1970s. The inclusion of dummies to take into account the Nixon price controls does not appear to alter these findings. The estimation does place a significant weight on a positive α in (1), and the hypothesis that $\alpha = 0$ is rejected. In other words, a pure accelerationist specification of the Phillips curve does not fit the Greenbook inflation forecasts for this period as well as a specification that gives some weight to a proxy of the long-run inflation goal. It is still worth noting, however, that if one were to fit a pure accelerationist specification to the Greenbook forecasts for this period, the estimated $\bar{u}_t$ would be qualitatively similar, the only difference being that the estimated natural rate at the beginning of the decade would be somewhat lower, at about 4 percent. Still, by the end of 1975, $\bar{u}_t$ is estimated at 6 percent. As an additional check on the qualitative findings for this period, we split the seventies into the years 1970-73 and 1974-79. We then estimate (1) with no time-varying intercept, backing out an average value of $\bar{u}$ for the two samples. Over the years 1970-73, $\bar{u}$ is estimated near 5 percent. In the subsequent years the estimate rises to roughly 7 percent.

In the early to mid-1980s, depicted in figure 2, $\bar{u}_t$ hovers around 7 percent. The standard deviation of the last observation for $\bar{u}_t$ in the sample is 0.7, indicating a nontrivial amount of imprecision in the

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[8] See, for example, Gordon (1997), Laubach (2001), and Staiger, Stock and Watson (1997).

[9] In the 1970s, the Greenbook was published at monthly frequency.

[10] In order to facilitate comparison with later estimates, the figures report estimates from one Greenbook per quarter. We take the earliest Greenbook available in each quarter.

estimate. There continues to be a significant role for long-run inflation expectations in (1). In the second half of the 1980s (figure 3), the estimated natural rate of unemployment declines to about 6 percent. It then increases somewhat in 1993-94, before starting to decline again. In the late 1990s (shown in figure 4), $\bar{u}_t$ declines noticeably and stabilizes around 4.5 percent in the 2000s. In this latest sample period, the estimated slope β is considerably smaller in absolute value than in the earlier samples, in accordance with those studies documenting a flattening of the Phillips curve.[11] The standard deviation of the last observation for $\bar{u}_t$ in the sample is around 0.30, which is similar to the value estimated at the end of 1996.

The estimates for $\bar{u}_t$ that we have shown have been computed by fixing the standard deviation for the innovation v_u. Given the random-walk assumption for the evolution of $\bar{u}_t$, changing $\sigma(v_u)$ impacts mainly the high-frequency volatility of the estimated $\bar{u}_t$, but its lower-frequency movements remain qualitatively the same as in figures 1-4.

Information about the Board staff assumptions regarding the natural rate of unemployment is not readily available for the 1970s and 1980s, but it is for the most recent period.[12] Coverage from 1989 to 1997 is irregular, but the Board's assessment of the evolution of the natural rate of unemployment over this period is fairly consistent with our estimates. For the post-1997 period, our estimates are near 4½ percent, which is consistently below the reported Board staff's assessment of 5 percent. Over the post-2002 period, however, the Board staff estimates of the natural rate did not include temporary productivity effects which were allowing a lower rate of unemployment to be consistent with stable inflation. These productivity effects likely contributed to lower the effective natural rate of unemployment implied by the Greenbook's inflation forecast.

3. *Real-time Estimates of the Natural Rate of Unemployment*

We now turn to consider estimates of the natural rate of unemployment based on two very stylized frameworks. In order to infer $\bar{u}_t$, we consider information available only up to time *t*. In this respect, our approach mimics as much as possible a real-time inference exercise. We do so because in order to

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[11] See, for example, Tetlow and Ironside (2007).

[12] This information can be found at the Federal Reserve Bank of Philadelphia Real-Time Data Research Center: http://www.phil.frb.org/research-and-data/real-time-center/greenbook-data/nairu-data-set.cfm.

have a fair comparison between the estimates of $\bar{u}_t$ obtained in this section with the Greenbook's estimates shown in the previous section, one needs to take into account the fact that at each Greenbook date the Board staff had to draw inference about $\bar{u}_t$ with information available only up to that date.

The simple models considered in this section draw inference about the natural rate of unemployment jointly from the dynamics of inflation and real economic variables. While signals about $\bar{u}_t$ are not based solely on inflation, we believe that it is still important that the estimates of $\bar{u}_t$ that we back out from this exercise maintain some reference to the evolution of inflation. After all, the notion of a natural rate of unemployment is intimately linked to the dynamics of inflation.

3.1 *Joint Estimation of the Natural Rate of Unemployment from the Phillips Curve and the IS Curve*

The first setup we consider to estimate the natural rate of unemployment is based on a Phillips curve and a simple reduced-form *IS* equation. The Phillips curve takes the form:

$$(4) \qquad \pi_t = \left(\alpha \bar{\pi}_{t-1} + (1-\alpha) \pi_{4,t-1} \right) - \sum_{i=1}^{2} \beta_i (u_{t-i} - \bar{u}_{t-i}) + x_{t-1} \delta + \varepsilon_{\pi,t}.$$

In essence, we maintain the same relationship used in the previous section to fit the Greenbook forecasts of inflation, though here we are fitting actual inflation. Variable definitions are the same as before, with π_t denoting the quarterly inflation rate at an annual rate.[13] We drop the superscript "*RT*" for real-time because to construct the variables we use the current vintage of data. However, the specific series we choose for our analysis are usually subject to only minor revisions involving seasonal adjustments. As a result, using the latest vintage should still retain the most salient features of the real-time vintages. As before, inflation expectations enter the Phillips curve as a weighted-average between trend inflation and recent inflation developments. A relatively small value for α should better capture inflation dynamics in the earlier part of the sample. A larger weight on trend inflation is instead associated with more recent inflation dynamics.

If one considers the Phillips curve relationship (4) alone, the estimated evolution of the unemployment rate gap, and thus of $\bar{u}_t$, depends only on the dynamics of inflation, trend inflation, and the supply

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[13] We use one-quarter inflation as the dependent variable rather than four-quarter inflation to avoid serial correlation in the error term.

shocks. It is possible to bring other data to bear on the estimation of $\bar{u}_t$ by specifying an additional equation that features a role for the unemployment rate gap. We do so here by specifying a simple reduced-form *IS* equation expressed in terms of the unemployment rate gap, which takes the form

$$(5) \qquad u_t - \bar{u}_t = \sum_{i=1}^{2} \rho_i (u_{t-i} - \bar{u}_{t-i}) + \sum_{i=1}^{2} \gamma_i (r_{t-i} - \bar{r}_{t-i}) + \varsigma rp_t + \varepsilon_{u,t}.$$

In the above equation, the unemployment rate gap depends on its own lags, lags of the real interest rate gap, $r_t - \bar{r}_t$, and a risk premium rp_t.[14] The equilibrium real interest rate is denoted by $\bar{r}_t$. We assume that both the natural rate of unemployment and the equilibrium real interest rate are time-varying and evolve as random walks:

$$(6) \qquad \bar{u}_t = \bar{u}_{t-1} + v_{u,t},$$

$$(7) \qquad \bar{r}_t = \bar{r}_{t-1} + v_{r,t}.$$

The innovations $v_{u,t}$ and $v_{r,t}$ are independent from the innovations in the measurement equations (4) and (5). The setup described by the relationships (4)-(7) shares some similarities with Laubach and Williams (2003).[15] It is, however, considerably simpler as Laubach and Williams focus on the output gap rather than the unemployment rate gap. As a result, their model imposes more structure, with the equilibrium real rate of interest affecting potential GDP growth and the output gap. In our framework, changes to the equilibrium real rate of interest can affect the unemployment rate gap via the *IS* equation (5). However, we do not model a relationship between the natural rate of unemployment and the equilibrium real rate of interest, and the shocks $v_{u,t}$ and $v_{r,t}$ are assumed to be uncorrelated.

Data

We use data at quarterly frequency. Inflation is measured by the core CPI. For the earlier part of the sample for which core CPI data is not available,[16] we measure inflation with the CPI index that excludes food. The unemployment rate is based on total civilian non-institutional population (16 years old and over). The trend inflation measure $\bar{\pi}_t$ is the same variable used in the previous section, to which we refer to for the discussion on how the variable is constructed pre-1980. Supply shocks in the Phillips curve (4) consist of two lags of the change in real oil prices and one lag of the Q4/Q4 change in real food

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[14] The risk premium is expressed in terms of deviations from its sample mean.

[15] See also Orphanides and Williams (2002).

[16] Core CPI data are available from 1957:Q1 on.

prices, where we use the CPI food price index. In the *IS* relationship (5), the real rate of interest is expressed as the difference between the nominal yield on corporate BBB bonds and $\bar{\pi}_t$. The risk premium is the difference between the BBB corporate yield and the yield on 10-year Treasuries.

Estimation

The Phillips and *IS* relationships in (4) and (5), together with the assumed evolution for $\bar{u}_t$ and $\bar{r}_t$ in (6) and (7), are estimated jointly by maximum likelihood using the Kalman filter. In order to capture potential changes in parameters other than $\bar{u}_t$ and $\bar{r}_t$, the estimation uses a rolling window of 72 quarters. The cost of using such a relatively short window is the magnification of small-sample bias at the estimation stage. While the size of the window which in this context would provide a good balance between benefits and costs at the estimation stage is not obvious, allowing for the possibility of structural change can be important when estimating the Phillips curve. As already mentioned, the weight of long-run inflation expectations and the slope of the Phillips curve have changed over time, and the same applies to the degree of pass-through of supply shocks to core inflation. [17]

The setup of the exercise is meant to approximate inference about the natural rate of unemployment in real time, where at each point in time t the most recent 72 quarters of available data are used to estimate (4)-(7) and evaluate $\bar{u}_t$. The first rolling window of estimation starts in 1955:Q1 and ends in 1972:Q4. As a result, the first real-time estimate of $\bar{u}_t$ is available for 1972:Q4.

When estimating a setup such as the one in (4)-(7), estimates of the standard deviation of the shocks to the permanent components, $\sigma(v_u)$ and $\sigma(v_r)$, can be biased toward zero. This so-called "pile-up" problem, whereby the Kalman filter underestimates the variance of the permanent component, has been discussed extensively in the literature. Failure to correct for this bias could lead to the wrong inference of too small of a variation in the natural rate of unemployment over time. To address this issue, we use Stock and Watson (1998) median unbiased estimator to obtain estimates of the signal-to-noise ratio for the natural rate of unemployment as $\lambda_u^2 \equiv \sigma^2(v_u)/\sigma^2(\varepsilon_\pi)(\beta_1^2 + \beta_2^2)$, and for the equilibrium real interest rate as $\lambda_r^2 \equiv \sigma^2(v_r)/\sigma^2(\varepsilon_u)(\gamma_1^2 + \gamma_2^2)$. [18] Since our approach uses a rolling window for estimating (4)-(7), we use the same rolling window to estimate the signal-to-noise ratios

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[17] See, for example, Fuhrer, Olivei, and Tootell (2009).

[18] The procedure uses functions of regression-based parameter stability test statistics, computed under the null hypothesis of no structural break.

over time.[19] These ratios are then imposed when estimating the other parameters of the model via maximum likelihood.

Results

Estimates of the evolution of the natural rate of unemployment from this exercise, together with the actual unemployment rate, are depicted in figure 5. At each point in time we consider, $\bar{u}_t$ is the last observation in the rolling estimation window. As a result, the filtered measure that we obtain is one-sided. According to this exercise, the natural rate of unemployment started to rise in the mid-1970s, reaching a peak in the late seventies. It then hovered near 6 percent in the eighties and early nineties, and then dropped to about 5 percent in the most recent sample period. It is interesting to note that the estimated $\bar{u}_t$ remains low in the most recent five years, despite the sharp rise in the unemployment rate. Estimation results (not reported) do conform to the notion that over time the estimated slope of the Phillips curve has decreased, while the weight placed on trend inflation in the Phillips curve's inflation expectations formation mechanism has increased.

3.2 *Joint Estimation of the Natural Rate of Unemployment from the Phillips Curve and the Beveridge Curve*

We now consider drawing inference about the natural rate of unemployment in the context of a Phillips curve and a Beveridge curve framework. Such a framework has been proposed by Dickens (2009) to exploit the Beveridge curve as an additional source of information about fluctuations in the natural rate of unemployment. The benefit of this approach is to bring information about the functioning of the labor market to bear on $\bar{u}_t$, while at the same time preserving the signals from inflation. Dickens shows that when complementing the Phillips curve with the Beveridge curve, the information coming from the Beveridge curve plays an important role in the determination of the dynamics of the natural rate of unemployment. Needless to say, the Beveridge curve is featured prominently in the current debate about the level of the natural rate of unemployment. This, to some extent, may reflect the fact that with a flatter Phillips curve, it is becoming more difficult to draw signals about the natural rate of

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[19] To offset some of the noise in the estimated ratios over the rolling moving window, we use a moving average of these estimates. If $\lambda^2_{i,[t-h,t]}$ is the signal-to-noise ratio estimated over the window $[t-h,t]$, when estimating (4)-(7) over the sample window $[t-h,t]$ we use as signal-to-noise ratios the centered moving-average of the nine point estimates of λ^2_i over the period $t-h-4$ to $t+4$.

unemployment from inflation dynamics. Still, the Beveridge curve has been a complement to the Phillips curve when assessing potential movements in the natural rate of unemployment also in past.[20]

In this setup, we keep the same Phillips curve relationship as in (4). Following Dickens, the Beveridge curve takes the form

$$(8) \qquad \ln\left(\frac{1-u_t}{u_t}\right) = c - a\bar{u}_t + b\ln(v_t / u_t) + \varepsilon_{v,t},$$

where v_t is the vacancy rate (the ratio of vacancies to the labor force). As previously, the natural rate of unemployment is assumed to evolve as a random walk, with innovations $v_{u,t}$ that are uncorrelated with the innovations $\varepsilon_{\pi,t}$ and $\varepsilon_{v,t}$ in (4) and (8), respectively. An important assumption underlying the unemployment-vacancies relationship in (8) is that all persistent shocks to the unemployment rate other than those acting through vacancies are shocks to the natural rate of unemployment. Such a simplification is controversial. The ongoing debate about the Beveridge curve, for example, highlights potential factors that, while shifting the curve, do not necessarily imply a change in the natural rate of unemployment.[21]

Data

We use the help-wanted index, with the scale adjustment suggested by Zargosky (1998), to characterize the vacancy series until 1997:Q4. From 2001:Q1 on, we use data from the Job Openings and Labor Turnover Survey (JOLTS). There is, therefore, a gap in the estimation as the help-wanted series became less informative with the growth of the Internet. Data sources for the other variables have already been mentioned.

Estimation

The Phillips curve, the Beveridge curve, and the law of motion for the natural rate of unemployment in (6), are estimated jointly via maximum likelihood using the Kalman filter. We assume that the shocks to the Phillips curve and to the Beveridge curve, $\varepsilon_{\pi,t}$ and $\varepsilon_{v,t}$, are uncorrelated. This implies that the only source of unpredictable co-movement between (4) and (8) stems from innovations to the natural rate of unemployment, which is captured by the intercepts in both equations. Because time variation in these

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[20] See, for example, Katz and Krueger (1999) for a discussion of movements in the natural rate of unemployment in the 1990s.

[21] For an overview of the recent debate about shifts in the Beveridge curve, see Daly, Hobijn, Sahin, and Valletta (2012).

intercepts is common to both measurement equations, the "pile-up" problem is attenuated (see Basishta and Startz, 2008), and the correction for bias applied in the previous exercises is not necessary. As before, in order to mimic inference about $\bar{u}_t$ in real-time, the estimation is performed over a rolling window of 72 quarters. The first real-time estimate of $\bar{u}_t$ is available for 1972:Q4.

Results

Estimates of the evolution of the natural rate of unemployment from this exercise, together with the actual unemployment rate, are depicted in figure 6. For the same reasons as in the previous exercise, the reported estimate of $\bar{u}_t$ obtained from our rolling estimation is always one-sided. The estimated values of the natural rate of unemployment are somewhat noisy, especially so in the 1980s. One notable feature of the estimated $\bar{u}_t$ is that it starts the sample period (the early 1970s) relatively high, at 6 percent. Such a level is similar to the estimates reported in Dickens.[22] The estimated natural rate of unemployment then climbs to about 7 percent by the end of the 1970s. It then stays between 6.5 and 7 percent for most of the eighties. From the late 1980s to 1997, the estimated $\bar{u}_t$ averages about 6 percent. As already mentioned, we lack data for the years 1997 to 2002. In the most recent period, contrary to the estimates obtained via the Phillips curve – *IS* curve framework, the natural rate of unemployment is estimated to have risen, with the 2012:Q4 reading at 6½ percent.

4. *The Greenbook and the Real-Time Estimates of the Natural Rate of Unemployment: A Comparison*

Figure 7 compares the Greenbook estimates of the natural rate of unemployment with the estimates obtained in the previous section. All of the estimates show an increase in the natural rate of unemployment in the 1970s. The Greenbook estimates increase earlier than the estimates derived from the Phillips curve – *IS* model (PC-*IS* henceforth). They instead catch up to the estimates derived from the Phillips curve – Beveridge curve model (PC-BC henceforth). Still, convergence between these two estimates is achieved by mid-1974. In the 1980s, there is a broad correspondence between the Greenbook and the PC-BC estimates. The estimates obtained from the PC-*IS* relationships, instead, are noticeably lower. From the late 1980s to the late 1990s, the three estimates are quite similar. One possible reason for this similarity is that during that period the Federal Reserve staff approach to estimating the natural rate of unemployment was relying heavily on the signals coming from a Phillips

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[22] See Dickens (2009), figure 4.4.

curve similar to the one in equation (4). In the most recent years for which Greenbook data is publicly available, the three estimates of the natural rate of unemployment are all low and near 5 percent, with relatively minor differences.

Overall, the Greenbook estimates of the natural rate of unemployment appear to share the same lower-frequency properties of the estimates obtained in real-time from the two simple specifications considered in the previous section. At a higher frequency, there are some differences, and the question then becomes whether the information contained in the estimates of the natural rate of unemployment from the two simple models has useful informational content. To address this question, we assess the extent to which the Greenbook forecasts of inflation would have been improved by using the PC-*IS* or the PC-BC estimates of the natural rate of unemployment. To this end, we estimate the following equation:

$$(9) \qquad e_t^4 \equiv \pi_{t+4}^4 - \pi_{t+4}^{4,GB} = \alpha_0 + \alpha_1(u_t - \overline{u}_t^{GB}) + \alpha_2(\overline{u}_t^{GB} - \overline{u}_t^{J}) + \eta_{t+4} .$$

The dependent variable in (9) is the Greenbook forecast error of four-quarter inflation. In addition to a constant, the explanatory variables are given by the Greenbook estimated measure of the unemployment rate gap, and by the difference between the Greenbook estimate of the natural rate of unemployment and the estimate obtained from either the PC-*IS* or the PC-BC models. The Greenbook estimate of the natural rate of unemployment is denoted by $\overline{u}_t^{GB}$. The notation $\overline{u}_t^{J}$ stands for either of the two other models (PC-*IS* or PC-BC). The disturbance η_{t+4} follows a moving-average process of order 3. We are interested in knowing whether divergences in the estimates of the natural rate of unemployment, $\overline{u}_t^{GB} - \overline{u}_t^{J}$, help to explain Greenbook inflation forecast errors.

In equation (9), we control for the Greenbook estimate of the unemployment rate gap. We do so because it is of some relevance to ascertain whether at the time the inflation forecast was made, the unemployment rate gap was factored efficiently into the forecast. We already know from the exercise in section 2 that Greenbook forecasts of inflation have consistently been driven, among other factors, by the unemployment rate gap. The estimated sensitivity of inflation to the unemployment rate gap, captured by the coefficient β in (1), has changed over time but has remained significant. Controlling for the unemployment rate gap in (9) provides an assessment of the extent to which the Federal Reserve staff's assessment of the trade-off between inflation and unemployment was appropriate.

Table (1) provides the results from estimating equation (9). Panel *A* considers the full sample from 1970 to 2007, while panel *B* excludes the 1970s from the estimation. The first column, in addition to the

estimate for the constant α_0, includes the Greenbook estimate of the unemployment rate gap only. The second column includes as an additional explanatory variable $\overline{u}^{GB} - \overline{u}^{J}$, where $\overline{u}^{J}$ is derived from the PC-*IS* model. In the third column, $\overline{u}^{J}$ is derived from the PC-BC model. As concerns the full sample results, there is only modest evidence that the Greenbook did not factor the estimate of the unemployment rate gap efficiently into the inflation forecast. The negative estimated coefficient implies that the Greenbook *under*stated the importance of the gap in driving actual inflation. When the estimated unemployment rate gap was positive, the Greenbook was forecasting a higher one-year-ahead inflation than actual, and vice versa with a negative unemployment rate gap. Such a finding has the following implications. If the Federal Reserve staff was estimating the unemployment rate gap correctly, the estimated negative coefficient implies that the slope of the Phillips curve β in (1) was being understated. But the negative estimated coefficient in the table could also mean that the Federal Reserve staff was mismeasuring the natural rate of unemployment, and the estimates that control for the difference $\overline{u}^{GB} - \overline{u}^{J}$ go in the direction of addressing this issue.

When including $\overline{u}^{GB} - \overline{u}^{J}$ in the regression, a negative sign for α_2 implies that when the Greenbook estimate of the natural rate of unemployment was higher than $\overline{u}^{J}$, the Greenbook forecast of inflation was higher than actual, and vice versa. The full-sample results indicate some significance of the correction to the Greenbook estimate of the natural rate of unemployment using the estimate obtained from the PC-BC model. The correction using the estimate of the natural rate of unemployment from the PC-*IS* model, is instead small and insignificant.

All of the reported findings for the full sample, however, are driven by the observations in the 1970s and change considerably when we consider only the 1980 to 2007 period. As panel *B* in the table shows, there is now little suggesting that the Greenbook estimate of the unemployment rate gap was factored inefficiently into the forecast of inflation. The difference between the Greenbook and the PC-*IS* estimates of the natural rate of unemployment becomes significant, while the correction from the PC-BC estimate is now insignificant. It should be noted that the standard errors reported in the table understate the uncertainty surrounding the estimated slopes in equation (9), as the explanatory variables are generated regressors. Moreover, the forecast improvement from correcting the Greenbook estimate of the natural rate of unemployment with the estimate obtained from the PC-*IS* model is small. This is illustrated in figure (8), which shows actual four-quarter inflation, the Greenbook

predicted value, and the Greenbook predicted value with the adjustment to the natural rate of unemployment obtained from the PC-*IS* model using the estimates in (9) for the 1980-2007 period.

We interpret the results as indicating that there is little evidence that the real-time estimates of the natural rate of unemployment obtained from the two models in the previous section would have consistently and meaningfully improved upon the Greenbook estimates in terms of forecasting inflation. As concerns the 1970s, the regressions in table 2 show a potentially larger economic impact from using the natural rate of unemployment estimated by means of the Phillips curve and the Beveridge curve.[23] According to the estimated α_2, a one percentage point difference between the Greenbook estimate of the natural rate of unemployment and the estimate derived from the PC-BC model, would have affected the inflation forecast by roughly 2 percent. The robustness of such a finding, however, is questionable. The significance of the adjustment to the PC-BC natural rate estimate diminishes once the regression controls for realized food inflation shocks, as the second column shows. Moreover, exclusion of the period 1973:Q3 to 1974:Q4 (in column 3), when oil price shocks were taking place, further reduces the importance of the adjustment. Overall, we interpret these findings as suggesting that the improvement to the Greenbook forecast from considering the PC-BC estimate of the natural rate of unemployment is concentrated within a few quarters, and its identification is complicated by the presence of supply shocks. Except for these few quarters, there is little evidence of a meaningful contribution from the adjustment.

5. *Concluding Remarks*

In this paper we have compared Greenbook estimates of the natural rate of unemployment to real-time estimates obtained from simple benchmark specifications. While there are differences across estimates, the estimates also share many similarities. There is little suggesting that the Greenbook estimates of the natural rate of unemployment lag systematically behind the real-time estimates. When differences arise, we find little compelling evidence that the real-time measures we have considered would have led to systematic improvements in the Greenbook inflation forecast.

While the paper is concerned with a real-time assessment of the natural rate of unemployment, it is worthwhile to briefly comment on estimates of the natural rate of unemployment obtained with information not available in real time. These estimates, too, are generally surrounded by a high degree

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[23] The contribution to the forecast error from the estimates of the natural rate of unemployment from the PC-*IS* model is not reported in this table, as it is not meaningful.

of uncertainty. An interesting feature of some of these estimates, however, is that they exhibit less volatility than the Greenbook estimates. This applies, for example, to the CBO estimate of the NAIRU, which we depict in figure (9) against the estimate of the natural rate of unemployment that we back out from the Greenbook. The low-frequency movements in the series are approximately the same, but fluctuations in the CBO NAIRU occur within a much smaller range. Is there evidence that this less volatile measure would have served policy better? Table 3 repeats the simple exercise of the previous section, where this time the variable of interest is the difference between the Greenbook estimate of the natural rate of unemployment and the CBO NAIRU. For the full sample estimates reported in column (1), there is some indication that the adjustment leads to an improvement in the forecast. For the 1970 to 1979 period, there is evidence of an economically meaningful adjustment from using the retrospective measure, though the estimated effect is not significant. However, similar to the results reported in Table 2, the economic and statistical significance of all of the estimated coefficients in the regression (including the constant) is highly sensitive to controlling for supply shocks.[24] There is some indication of an improvement in the Greenbook inflation forecast during the post-1980s sample, but similar to the results obtained with the real-time estimates of the natural rate of unemployment from the PC-*IS* model, the improvement is small. Too, as the estimates in column 4 indicate, the improvement is concentrated in the first-half of the 1980s.

This lack of a significant improvement to the Greenbook inflation forecast should caution against concluding that the FOMC should have strived for a lower rate of unemployment whenever the CBO estimate of the NAIRU happened to be below the Greenbook estimate of the natural rate, and vice versa. After all, even the CBO estimate of the NAIRU is uncertain, and the lack of a clear advantage of this measure on the inflation forecast front is most likely a reflection of such an uncertainty. Still, a caveat is in order. If inflation and unemployment are strictly related only when the unemployment rate is either very high or very low, then thinking about the lowest achievable unemployment rate with stable inflation is relevant given the Federal Reserve's dual mandate. While there is some evidence supporting nonlinearities in the inflation-unemployment tradeoff,[25] the analysis and the conclusions drawn in this paper are based on a linear relationship between inflation and unemployment.

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[24] In particular, the same qualitative findings (with even less economic significance) reported in the second and third column of Table 2 apply. Results are available upon request.

[25] See, among others, Stock and Watson (2010).

Table 1

Predictability of Greenbook Inflation Forecast Errors: 1970 to 2007, and 1980 to 2007

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A. 1970 to 2007

Constant	.148	.127	.097
	(.178)	(.152)	(.141)
$u_t - \bar{u}_t^{GB}$	-.267*	-.261*	-.339*
	(.122)	(.123)	(.142)
$\bar{u}_t^{GB} - \bar{u}_t^{PC,IS}$		-.099	
		(.184)	
$\bar{u}_t^{GB} - \bar{u}_t^{PC,BC}$			-1.262*
			(.504)

B. 1980 to 2007

Constant	-.206**	-.163**	-.230**
	(.074)	(.059)	(.081)
$u_t - \bar{u}_t^{GB}$	-.067	-.075	-0.062
	(.064)	(.063)	(.065)
$\bar{u}_t^{GB} - \bar{u}_t^{PC,IS}$		-.306**	
		(.093)	
$\bar{u}_t^{GB} - \bar{u}_t^{PC,BC}$			-.169
			(.154)

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Standard errors are in parentheses. * and ** denote significance at the 5 percent and 1 percent level, respectively.

Table 2

Predictability of Greenbook Inflation Forecast Errors: 1970 to 1979

_____

	1970 to 1979	1970 to 1979 with food price shocks	excluding 1973:Q3 to 1974:Q4
Constant	.541	.569	.382
	(.326)	(.334)	(.292)
$u_t - \bar{u}_t^{GB}$	-.770**	-.707*	-.360*
	(.256)	(.274)	(.161)
$\bar{u}_t^{GB} - \bar{u}_t^{PC,BC}$	-2.110**	-1.384	-0.101
	(.683)	(.998)	(.692)

_____

Standard errors are in parentheses. * and ** denote significance at the 5 percent and 1 percent level, respectively.

Table 3

Predictability of Greenbook Inflation Forecast Errors: 1970 to 2007 and 1980 to 2007

	1970 to 2007	1970 to 1979	1980 to 2007	1986 to 2007
Constant	.283	1.04**	-.094	-.085
	(.196)	(.356)	(.055)	(.057)
$u_t - \overline{u}_t^{GB}$	-.302*	-.894*	-.082	.025
	(.136)	(.277)	(.062)	(.093)
$\overline{u}_t^{GB} - \overline{u}_t^{NAIRU}$	-.634*	-.821	-.320**	-.140
	(.258)	(.508)	(.106)	(.095)

Standard errors are in parentheses. * and ** denote significance at the 5 percent and 1 percent level, respectively.

Figure 1
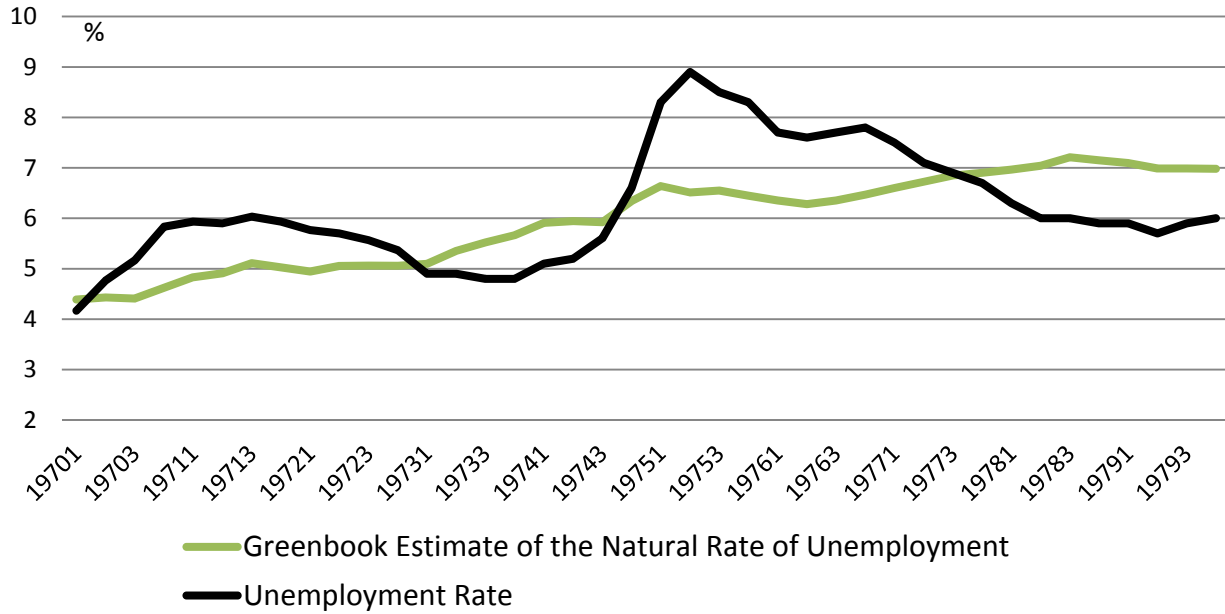## Greenbook Estimate of the Natural Rate of Unemployment:
1970 to 1979

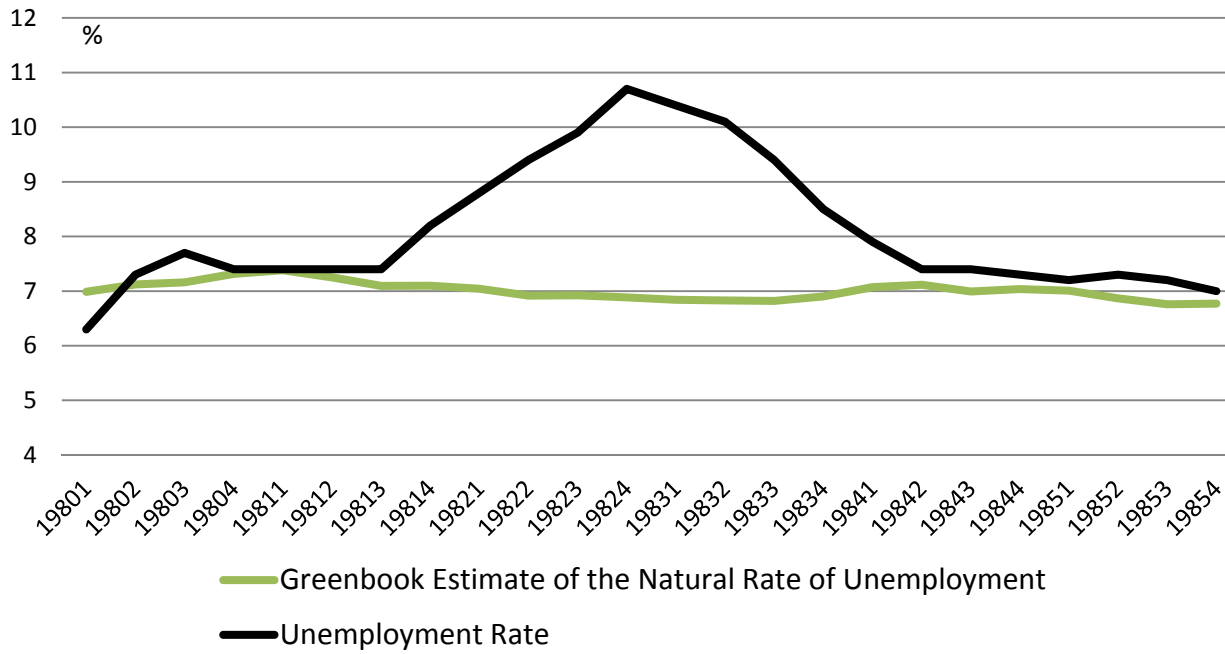— Greenbook Estimate of the Natural Rate of Unemployment

— Unemployment Rate

Figure 2
Greenbook Estimate of the Natural Rate of Unemployment:
1980 to 1985

— Greenbook Estimate of the Natural Rate of Unemployment

— Unemployment Rate
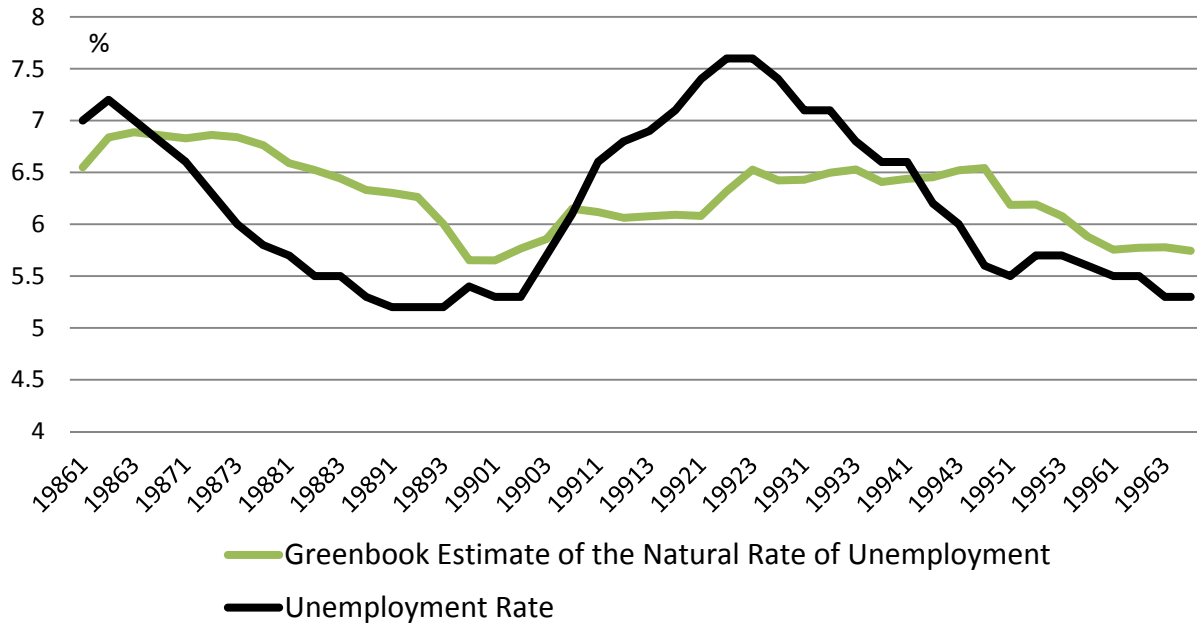
Figure 3
Greenbook Estimate of the Natural Rate of Unemployment:
1986 to 1996

Figure 4
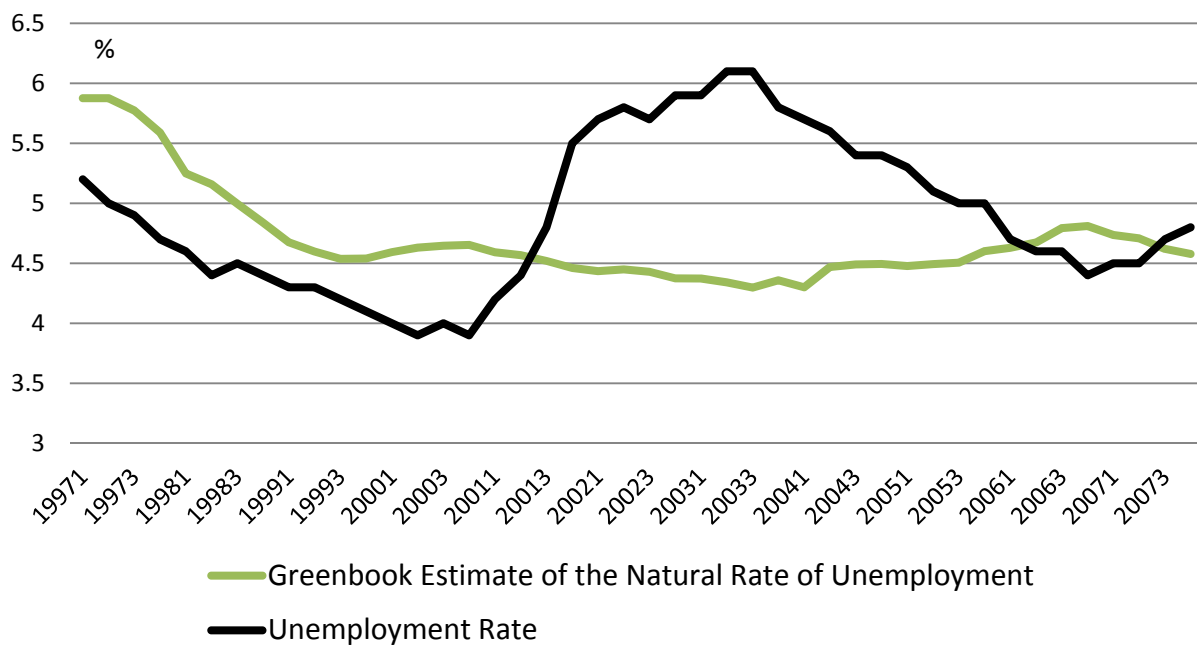## Greenbook Estimate of the Natural Rate of Unemployment:
1997 to 2007

Figure 5
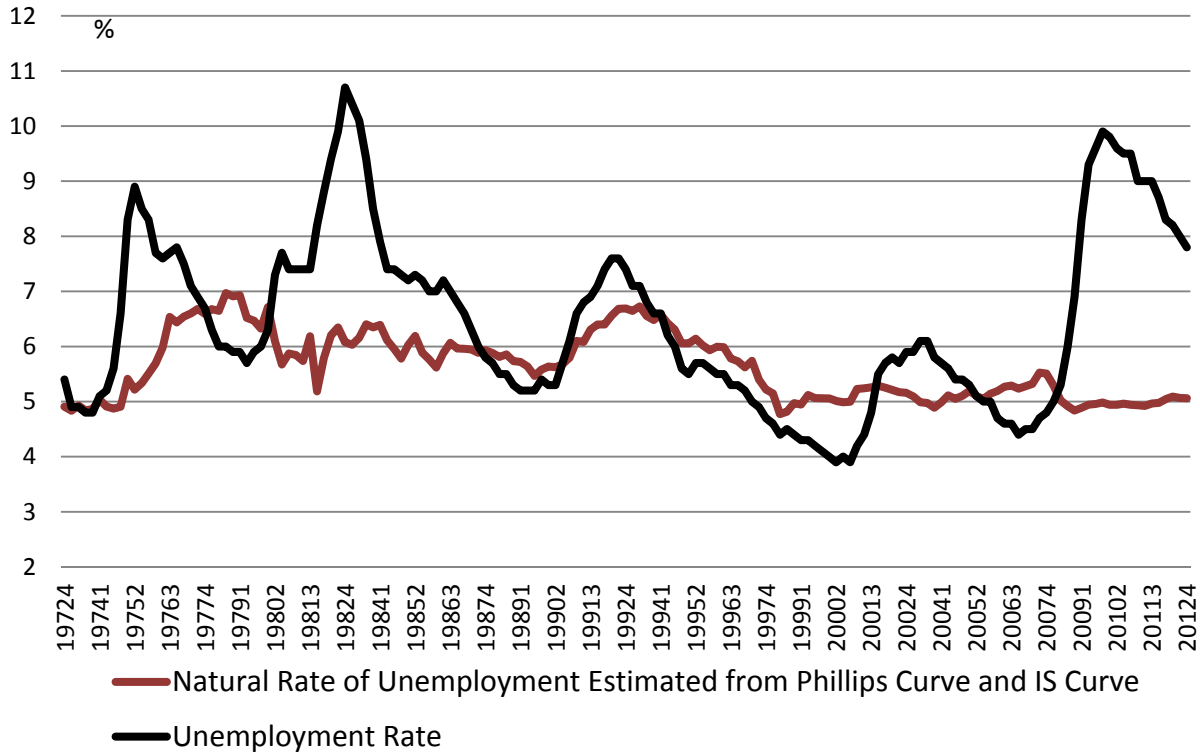## Natural Rate of Unemployment Estimated from Phillips Curve and IS Curve

Natural Rate of Unemployment Estimated from Phillips Curve and IS Curve

Unemployment Rate

Figure 6
Natural Rate of Unemployment Estimated from Phillips Curve and Beveridge Curve

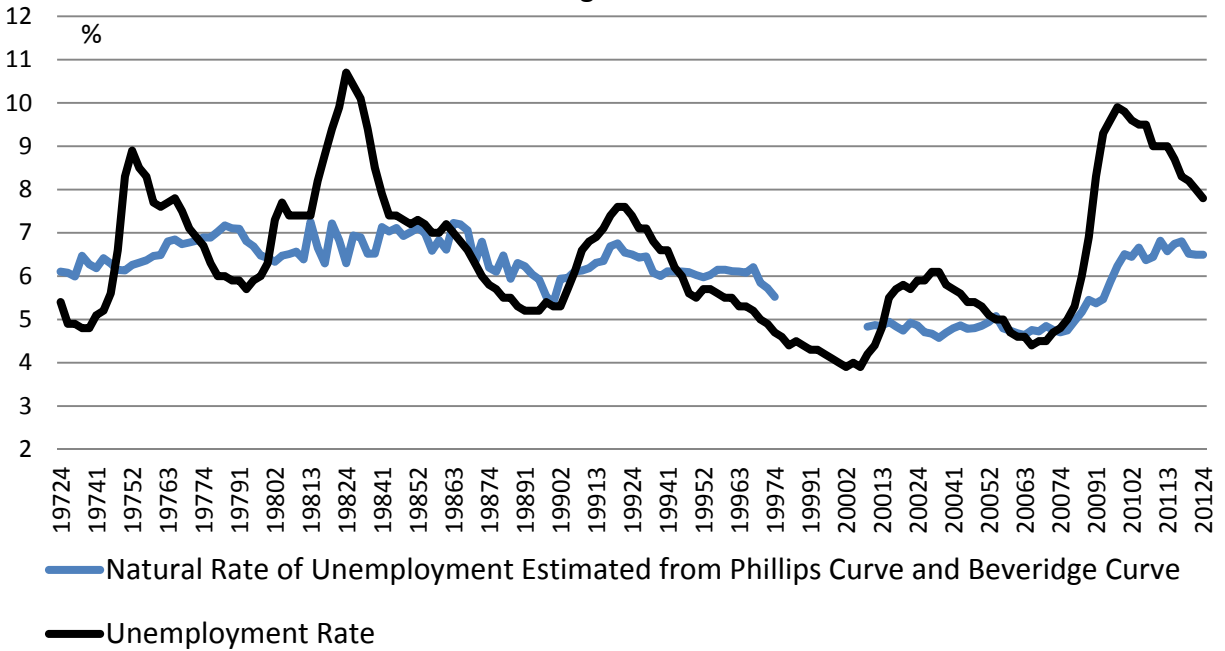Natural Rate of Unemployment Estimated from Phillips Curve and Beveridge Curve

Unemployment Rate

Figure 7
Real-Time Estimates of the Natural Rate of Unemployment

Greenbook Estimate of the Natural rate of Unemployment

Natural Rate of Unemployment Estimated from Phillips Curve and IS Curve

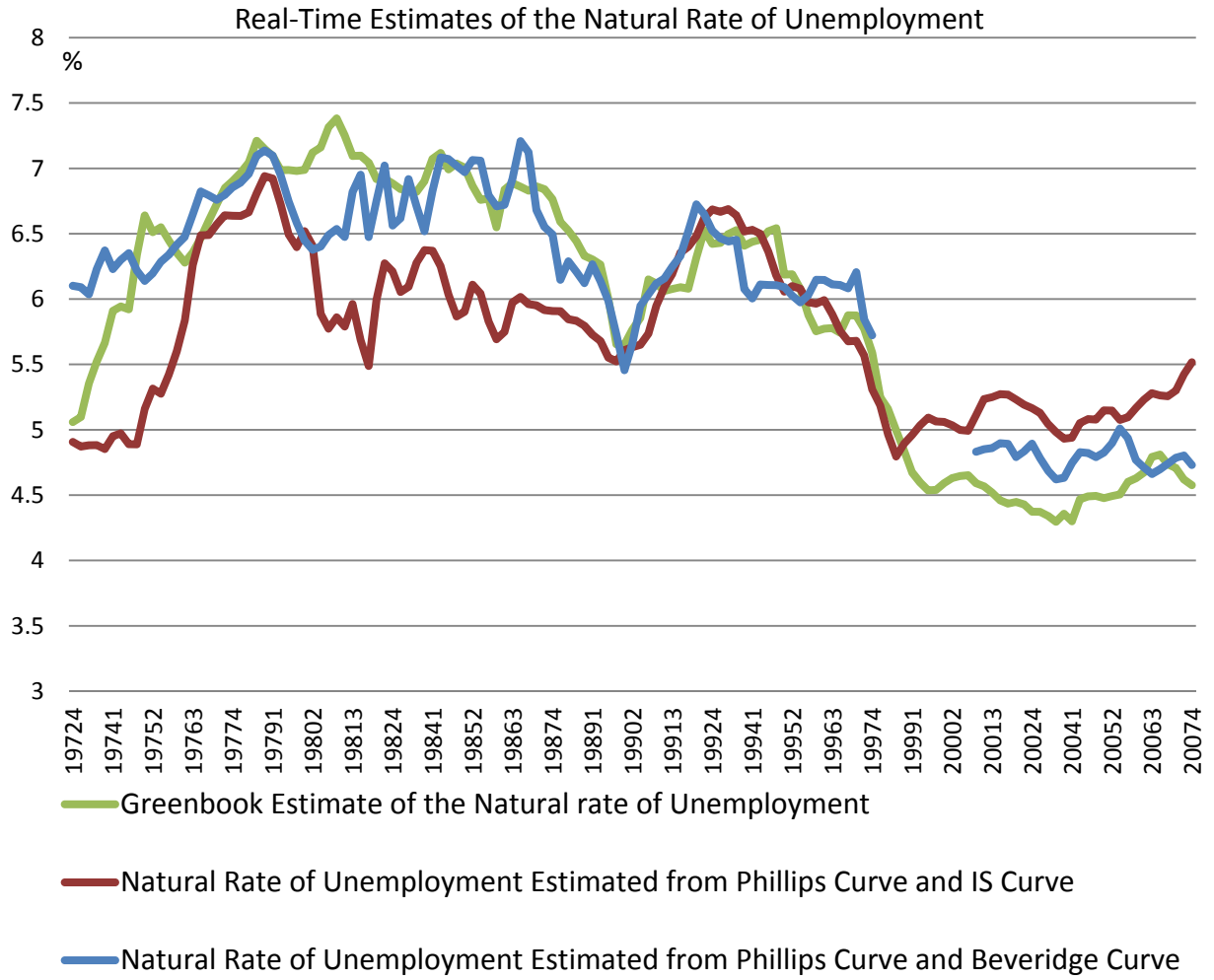Natural Rate of Unemployment Estimated from Phillips Curve and Beveridge Curve
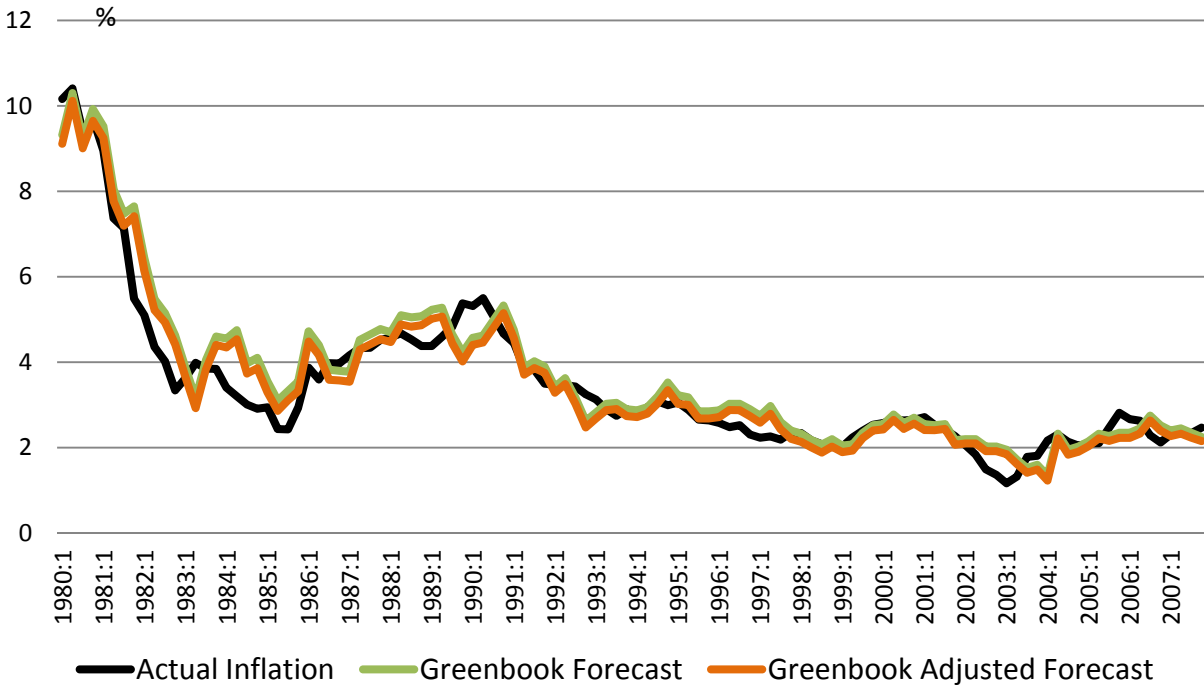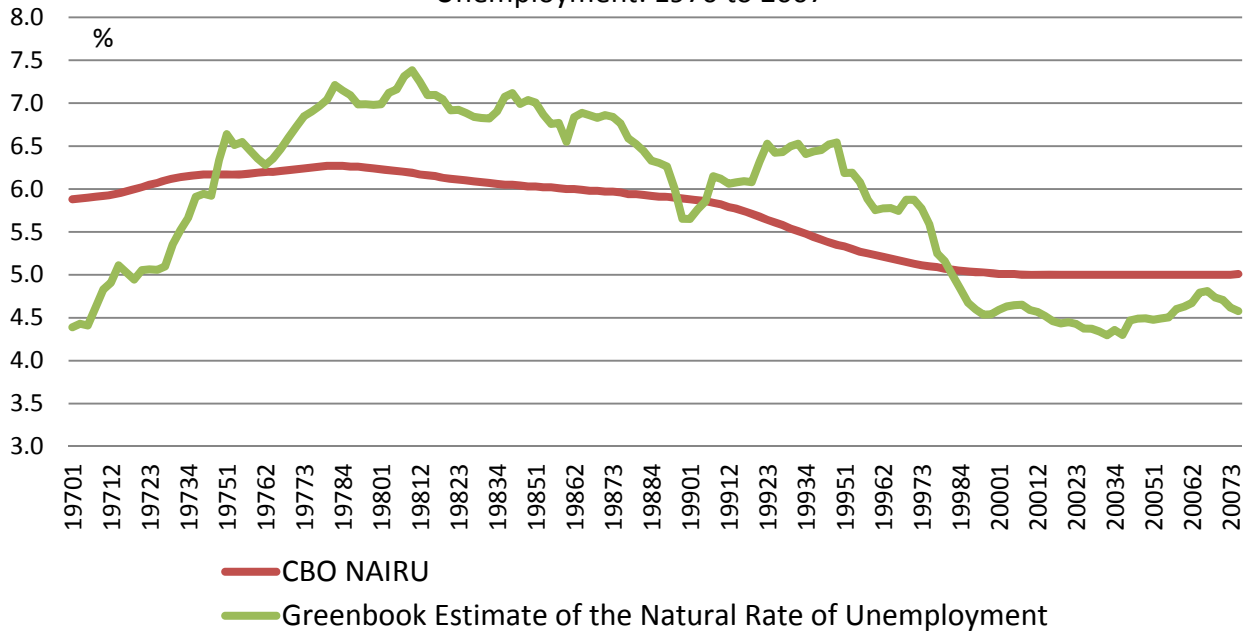
Figure 8
Actual and Predicted Core CPI Inflation

Legend: Actual Inflation — Greenbook Forecast — Greenbook Adjusted Forecast

Figure 9
CBO NAIRU and Greenbook Estimate of the Natural Rate of Unemployment: 1970 to 2007

Legend: CBO NAIRU — Greenbook Estimate of the Natural Rate of Unemployment

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