Federal Reserve Tools for Managing Rates and Reserves

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1 Introduction

This paper studies new monetary policy tools for managing short-term market rates. The tools we consider are interest on excess reserves (IOER), reverse repurchase agreements (RRPs) with a wide range of market participants, and the term deposit facility (TDF).

The Federal Reserve responded to the 2007-09 financial crisis and its aftermath with a variety of monetary policy measures that dramatically increased the supply of reserves. In part, this has led the federal funds rate, and other money market interest rates, to be somewhat more variable than before. In October 2008, the Federal Reserve began paying IOER to depository institutions (DIs); despite this, money market rates have consistently remained below the IOER rate. In June of 2011, the Federal Open Market Committee (FOMC) announced new tools, term RRPs and the TDF, aimed at managing short-term market interest rates and keeping them close to the IOER.[2] In August 2013, the FOMC also announced a fixed-rate, full-allotment overnight RRP (ON RRP) as another potential tool.[3]

The sets of institutions that have access to each tool vary. IOER is paid to DIs holding reserve balances at the Federal Reserve. [4] Similarly, the TDF is a facility offered to DIs, who are eligible to earn interest on balances held in accounts at the Federal Reserve, that allows them to hold deposits for longer term for an interest rate generally exceeding the IOER. In contrast, term and ON RRPs are available to a range of bank and non-bank counterparties, giving them the opportunity to make the economic equivalent of collateralized loans to the Federal Reserve.[5] An institutional background and explanation of these tools is provided in Section 2.

We develop a general equilibrium model to study how the Federal Reserve can use its tools to manage short-term interest rates and the large level of reserves on its balance sheet. Our model extends Martin, McAndrews, and Skeie (2013) (henceforth referred to as MMS) to include two separate banking sectors, liquidity shocks occurring in an interim period, interbank lending frictions, and bank moral hazard. The model provides a framework within which to study the effectiveness of the Federal Reserve tools in supporting interest rates and delivers insight into the economic mechanisms that determine equilibrium rates and quantities.

[2] Source: http://www.federalreserve.gov/monetarypolicy/files/fomcminutes20110622.pdf

[3] Source: http://www.federalreserve.gov/newsevents/press/monetary/20130821a.htm

[4] IOER differs from interest on reserves (IOR) in that IOER is paid to reserve holdings in excess of the reserve requirement.

[5] http://www.newyorkfed.org/markets/expandedcounterparties.html provides a list of the counterparties.

The framework allows for addressing several questions about the optimal provision of public money, such as reserves, and private money, such as bank deposits. In particular, how do reserves differ from government bonds as a source of public money and other types of public assets created by the Federal Reserve? How does public money differ from private money provided by bank deposits? What is the welfare benefits and costs of public and private money-like assets? What is the optimal quantity of public versus private money? Specifically, what is the optimal level of central bank reserves held by banks? Finally, what is the optimal composition of different types of public versus private money, and liquid money versus illiquid assets?

Banks in our model face two main frictions. First, they have the ability to risk-shift returns on assets. So that banks do not risk-shift, regulators must impose a leverage ratio on banks, so that they hold an appropriate amount of equity on their balance sheet. Equity can be costly for many reasons, in particular if investors have a natural preference for liquidity. The cost of having to hold additional equity is captured in a lower deposit rate. Second, banks face interbank lending frictions in the form of monitoring costs when they borrow in the interbank market. The interbank lending frictions lead to higher rates on larger interbank loans.

We assume that there are two banks.[6] At date 1, one of the banks may face a "liquidity shock", which we model as a withdrawal by depositors to purchase government bonds. This shock reduces the amount of deposits at the bank experiencing the shock and increases deposits at the other bank. The bank receiving additional deposits faces an unplanned balance sheet expansion and requires more capital, due to the risk-shifting incentive. This will lead the bank to lower its deposit rate. In equilibrium, the liquidity shock leads to downward pressure on both deposit rates and government bond yields.

Liquidity shocks affect banks' asset returns as well. When a bank faces stochastic withdrawals by its depositors, liquid reserves serve as a buffer, allowing the bank to fund these withdrawals with accumulated reserves. If the bank does not have enough reserves, it can borrow in the interbank market, which is costly because of the interbank lending friction. Hence, the liquidity shock gives banks an incentive to hold their assets as liquid reserves, rather than tie them up in illiquid assets such as loans to firms. For illiquid assets to be held in positive amounts, they must earn a premium over reserve holdings in equilibrium.

The central bank has the ability to issue public money in the form of reserves or RRPs. If banks hold sufficiently many reserves, they do not need to borrow in

[6] For simplicity, we will refer to DIs as "banks" in our framework.

the interbank market in case of a liquidity shock, reducing the interbank market friction. However, reserves can only be held by banks, and a large supply of reserves can lead to increased bank balance sheet size. Balance sheet expansion is costly when incentives to risk-shift are increasing in balance sheet size, as in our model. In contrast, RRPs can be held by non-banks as well as banks. As more non-banks hold RRPs, the balance sheet of the banking sector decreases, reducing the balance sheet cost. Because overnight interest rates reflect daily liquidity shocks, the fixed-rate, full-allotment ON RRP is the most effective facility for setting a fixed reservation rate for those intermediaries; term or fixed-quantity RRPs cannot achieve the same level and stability of interest rates.

In comparison to the RRP, the TDF absorbs liquid reserves without reducing the size of bank liabilities and increases bank asset returns more directly. If RRPs or the TDF are used in sufficiently large size, they can increase interbank market activity by reducing the size of liquid reserves used to ward off liquidity shocks and can raise equilibrium bank asset returns. We find that utilizing both the TDF and the RRP together may support rates most effectively if both bank risk-shifting incentives and interbank lending frictions are large enough and quickly increasing.

We also find that limited competition may be an additional reason for spreads between IOER and money market rates. In particular, limited competition gives banks more oppertunities to extract surplus from households. Monopolistic banks will offer a contract to households that makes them completely indifferent between storing wealth and holding deposits. Depending on model parameters, this can offer households expected returns that are significantly below the competitive contract and further below IOER. When banks are monopolistic within their sector but engage in Bertrand competition across sectors, many contracts are possible with the potential of some of them having equilibrium money market returns also below IOER and the competitive outcome. In either case, the tools can be used to raise rates and limit the amount of surplus extracted from the household.

Our paper fits broadly into the existing literature on the public supply of liquidity and money, monetary policy implementation, IOER, and reserves. Holmstrom and Tirole (1998) highlight the failure of private economies to effectively supply necessary quantities or quality of private liquidity to investors in the case of aggregate liquidity shocks. Our paper shows that public money supply is critical even without aggregate shocks. Poole (1970) shows that the effectiveness of policy based on targeting interest rates versus money stocks is not well determined and depends on parameter values, but policy using both is always weakly superior to either of the two used alone. Ennis and Keister (2008) provide a general framework for understanding monetary policy implementation with IOER. They show that IOER can help implement a floor on market rates and allows the Federal Reserve to manage interest rates. Bech and Klee (2011) analyze the federal funds market in the presence excess reserves. They argue that since government-sponsored enterprises (GSEs) do not have access to IOER, they have lower bargaining power and trade at rates lower than IOER, thus resulting in the observed IOER-federal funds effective rate spread.

Kashyap and Stein (2012) show that, with both IOER and reserve quantity control, the central bank can simultaneously maintain price stability and address externalities resulting from excessive bank short-term debt issuance. MMS focuses on the effects of excess reserves on inflation, interest rates, and bank credit. They find that these parameters are largely independent of bank reserve holdings unless external frictions are present. In particular, very large reserves can be contractionary in bank lending and be deflationary at the zero lower bound of interest rates. The current paper is the first to analyze the additional Federal Reserve tools and their effectiveness in controlling short-term money market rates and managing Federal Reserve liabilities. We provide positive-result predictions on the effects of the tools on a dispersion of interest rates and normative results that indicate the optimal quantity and composition of the provision of public and private money.

The paper is organized as follows: Section 2 explains institutional details on the Federal Reserve's monetary policy before, during, and after the 2007-09 financial crisis and provides descriptions of IOER, RRPs and the TDF. Section 3 presents and solves the benchmark model. Section 4 incorporates the RRP and the TDF into the benchmark model and analyzes their equilibrium results and effectiveness. Section 5 offers some brief extensions to monopolistic and oligopolistic banks. Section 6 concludes. Proofs of most propositions and figures are in the Appendix.

2 Institutional Background

Prior to the financial crisis of 2007-2009, the Federal Reserve closely controlled the supply of reserves in the banking system through its open market operations (OMOs). In an OMO, the Federal Reserve buys or sells assets, either on a temporary basis (using repurchase agreements) or on a permanent basis (using outright transactions), to alter the amount of reserves held in the banking system.[7] For

[7] Assets eligible for OMOs are Treasuries, agency debt, and agency mortgage-backed securities (MBS).

example, purchasing Treasuries will increase the amount of reserves in the system.

By adjusting the supply of reserves in the system, the open market trading desk (the Desk) at the Federal Reserve Bank of New York (NY Fed) could influence the level of the federal funds rate, the rate at which DIs lend reserves to each other. DIs in the US are required to maintain a certain level of reserves, proportional to specified deposit holdings, which, in addition to precautionary demand for reserve balances, creates a demand curve for reserves. The interest rate at which this demand and the supply curves intersect increases when the Desk reduces the supply of reserves, for example.[8] Through arbitrage, the level of the federal funds rate influences other short-term money markets rates.

In response to the 2007-09 financial crisis and subsequent economic downturn, monetary policy measures included large-scale lending to provide liquidity to financial institutions, and large-scale asset purchases through the large-scale asset purchase program (LSAP) to stimulate the economy by lowering longer term interest rates.[9] This facilitated a very large increase in the supply of reserves.[10] Moreover, in December 2008, the FOMC lowered the target federal funds rate to a range of 0 to 25 basis points, its effective zero bound, to help stimulate the economy.[11]

The effective federal funds rate, a weighted average of federal funds trades arranged by brokers, remained below 25 basis points, as shown in figure 1.[12] Figure 1 highlights that the federal funds rate fluctuated closely with other short-term money market rates, including the overnight Eurodollar rate and the overnight Treasury repo rate. These rates are seen to be typically decreasing in the level of reserves.

See figure 1

Following the LSAPs and the large expansion of the balance sheet, the Federal Reserve has been preparing a variety of tools to ensure that short-term rates can be lifted when needed. IOER has been used as one of these tools since October 2008; however two of these tools, RRPs with an extended range of counterparties, and the TDF, have not been implemented in large-value facilities as of yet. In a 2009 speech, NY Fed President William Dudley, referred to the RRP and the TDF as the "suspenders" that will support IOER, i.e. the "belt," in allowing the

[8] See Ennis and Keister (2008) and Keister, and Martin, and McAndrews (2008) for a more detailed introduction to traditional Federal Reserve monetary policy and OMOs

[9] The LSAPs are sometimes referred to as "quantitative easing" (QE)

[10] See Gagon, Raskin, Remanche, and Sack (2010) for more information on the LSAPs.

[11] Source: http://www.federalreserve.gov/monetarypolicy/files/fomcminutes20081216.pdf

[12] One common explanation for this is the currently large presence of Government Sponsored Enterprises (GSE) lending in the fed funds market. GSEs are not eligible for IOER and therefore tend to lend at rates below 25 basis points (see Beck and Klee (2011)).

Federal Reserve "retain control of monetary policy."[13] In August 2013, the FOMC announced potential use of an additional tool, the ON fixed-rate, full-allotment RRP.

2.1 Interest on Excess Reserves

To manage short-term rates in the face of large excess reserves, the Federal Reserve began to pay DIs IOER in October 2008. IOER differs from interest on reserves (IOR) in that IOER is paid to reserve holdings in excess of the reserve requirement. The Financial Services Regulatory Relief Act of 2006 originally granted the Federal Reserve the ability offer IOER. However, the original authorization was only applicable to balances held by DIs starting October 2011. The Emergency Economic Stabilization Act of 2008 accelerated the start date to October 2008.[14]

The interest owed to a balance holder is computed over a maintenance period, typically lasting one to two weeks depending on the size of the DI. Interest payments are typically credited to the holder's account about 15 days after the close of a maintenance period.[15] IOER was first offered in October of 2008 at 75 basis points, but is currently at 25 basis points where it has been since December 2008. Institutions that are not DIs are not eligible to earn IOER.[16]

2.2 Reverse Repurchase Agreements

An RRP is economically equivalent to a collateralized loan made to the Federal Reserve by a financial institution. RRPs have historically been used, though somewhat infrequently, by the Federal Reserve in the conduct of monetary policy, arranged with a set of counterparties called "primary dealers."[17]

In October 2009, the Federal Reserve announced that it was considering offering RRPs on a larger scale to an expanded set of counterparties.[18] The expanded set of counterparties include DIs as well as non-DIs, such as MMFs, GSEs, and dealers, increasing both the number and the type of Federal Reserve counterparties.[19] In

[13] Source:http://www.newyorkfed.org/newsevents/speeches/2009/dud090729.html

[14] Source:http://www.federalreserve.gov/newsevents/press/monetary/20081006a.htm

[15] Source:http://www.federalreserve.gov/newsevents/press/monetary/monetary20081006a2.pdf

[16] Source:http://www.federalreserve.gov/newsevents/press/monetary/20081006a.htm

[17] See http://www.newyorkfed.org/markets/primarydealers.html.

[18] Source:http://www.newyorkfed.org/newsevents/news/markets/2009/an091019.html

[19] A full list of current eligible counterparties is available at http://www.newyorkfed.org/markets/expanded_counterparties.html

addition, in August 2013 the Federal Reserve announced it would further study the potential for adopting a fixed-rate, full-allotment ON RRP facility.[20] In his September 2013 speech, President Dudly discussed this new facility as a way to support money market rates by allowing counterparties a flexible amount of investment at a fixed rate when needed.[21]

RRPs do not change the size of the Federal Reserve's balance sheet, but modify the composition of its liabilities. Indeed, each dollar of RRPs held by counterparties reduces one-for-one reserves held by DIs.[22]

The Federal Reserve Bank of New York has held numerous small-scale temporary operational exercises of RRPs for eligible counterparties starting in the fall of 2009.[23] Small-scale operational exercises held in April, June and August of 2013 were limited in terms of their overall size (less than $5 billion) and were focused on ensuring operational readiness on the part of the Federal Reserve, the tri-party clearing banks, and the counterparties.[24] While the Desk has the authority to conduct RRPs at maturities ranging from 1 business day (overnight) to 65 business days, the operational exercises thus far have typically ranged from overnight to 5 business days, with several of the August 2013 operational exercises consisting of overnight RRPs. Overnight RRPs were originally settled the day after auction; however overnight RRPs with same day settlement were offered starting in August 2013.

At the September FOMC meeting, the committee authorized the Desk to implement fixed-rate RRP exercises with per-counterparty bid caps to limit the aggregate size of the facility. In comparison to previous exercises, these exercises should better simulate the fixed-rate, full-allotment facility, which was discussed in the July 2013 FOMC minutes.[25]

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[20] Source:http://www.federalreserve.gov/monetarypolicy/files/fomcminutes20130731.pdf

[21] Source:http://www.newyorkfed.org/newsevents/speeches/2013/dud130923.html

[22] See the New York Fed page on RRPs for more information: http://data.newyorkfed.org/aboutthefed/fedpoint/fed04.html

[23] See the New York Fed page on temporary operations for a listing of recent RRP excercises: http://www.newyorkfed.org/markets/omo/dmm/temp.cfm

[24] These excercises were approved by the FOMC in November 2009. See:http://www.federalreserve.gov/monetarypolicy/files/fomcminutes20091216.pdf

[25] Source:http://www.newyorkfed.org/markets/rrp_faq.html

2.3 Term Deposit Facility

The TDF is another policy tool that can reduce reserves but it is available only to DIs.[26] The TDF was approved April 2010, following the approval of amendments to Regulation D (Reserve Requirements of Depository Institutions), allowing Federal Reserve Banks to offer term deposits to institutions eligible to earn interest on reserves.[27] Small value temporary operational exercises of term deposits have occurred since June 2010, and recent small value operational exercises have been held in March, May, and July, and September of 2013.[28]

As was the case for RRPs, the TDF does not change the size of the Federal Reserve's balance sheet, but alters the composition of its liabilities. Reserves used to finance purchases of term deposits are unavailable to DIs until the term deposit matures. The TDF therefore directly absorbs reserves when banks substitute reserve holdings for TDF holdings.

3 Benchmark Model

3.1 Agents

The economy lasts three periods $t = 0, 1, 2$ and consists of two sectors, $i = 1, 2$, which are partially segmented. Each sector contains three agents: a bank, a firm, and a household. In addition, a financial intermediary that we associate with an MMF operates across both sectors. The banks, the firms, and the MMF act competitively and are risk-neutral. There is also a central bank and the government that both issue liabilities across sectors but do not behave strategically. We consider an ex-ante symmetric case where the sectors, agents, and initial asset holdings are identical. By symmetry, returns for assets issued at date $t = 0$ will be equal across sectors in equilibrium.

At date 0, households in each sector receive an endowment (W) that can be held in the form of deposits, D^0, or equity, E^0, in the bank of their sector or in MMF shares (F^0). No other agent has an endowment.

The supply of reserves and government bonds are set exogenously and denoted by M and B, respectively. The interest paid on excess reserves, or IOER, is set exogenously and denoted by R^M paid each period, while the interest paid on government

[26] Source:http://www.federalreserve.gov/newsevents/press/monetary/20100430a.htm

[27] Source:http://www.federalreserve.gov/newsevents/press/monetary/20100430a.htm

[28] Source:http://www.frbservices.org/centralbank/term_deposit_facility_archive.html

bonds is determined in equilibrium and denoted R^B.

Banks take deposits from households and can invest them in loans to the firm from the same sector (L) or hold them as reserves at the central bank (M^0). Note that only banks can hold reserves. Reserves are injected into the economy by the central bank purchasing bonds, so the quantity of bonds held by the central bank, B^{CB}, is equal to the supply of reserves M. Firms borrow from banks and finance projects with a concave and strictly increasing production function, with marginal real return given by $r(L)$. The firms' output is sold as consumption goods to households at date $t = 2$.[29] The MMF can sell shares to households and invests in government bonds. We denote the MMF's bond holdings as B^H.

Banks, firms, and the MMF are profit maximizers, while households seek to maximize consumption. There are centralized markets for goods, bonds, and reserves, which imply they have common prices and returns across sectors. In contrast, deposits and bank equity have separate markets in each sector and their returns can vary across sectors. We abstract from credit risk for simplicity, as the focus of the paper is the use of monetary policy tools in a stable, non-crisis environment.

3.2 Timeline

At $t = 0$, the household of sector i deposits D^0 and holds E^0 of equity in the local bank and invests F^0 in MMF shares. Banks accept deposits, issue equity, hold reserves, and lend to firms at a rate R^L. The MMF sells shares and purchases bonds.

At $t = 1$, a liquidity shock hits one of the two sectors. The probability that sector i is hit is $\frac{1}{2}$ for $i = 1, 2$. The nature of the liquidity shock is that the household in the shocked sector demands an additional quantity of MMF shares equal to a fraction λ of its bank assets $(D^0 + E^0)$. The household receives an interest rate R^W on deposits withdrawn. [30] The household in the non-shocked sector can redeem MMF shares at an equilibrium price of P^B. The quantity redeemed is denoted by B^1. The revenue from MMF redemptions can be deposited in the bank of the same sector or invested in additional bank equity. New deposits are denoted D^1 and new equity investment is denoted E^1. For simplicity, we assume that households cannot deposit or hold equity in a bank if they have just withdrawn from the same bank in $t = 1$.

Banks can use reserves to meet withdrawals. Reserves receive a return of R^M, corresponding to the IOER, at each date $t = 1, 2$ per unit held in the previous

[29] Note that uppercase variables denote nominal values while lowercase variables denote real values. Also subscripts always represent the sector.

[30] We assume that equity cannot be withdrawn or sold but that λ is small enough so that $\lambda(D_i^0 + E_i^0) < D_i^0$.

period. If a bank does not have enough reserves, it can borrow I from the other bank in the interbank market at an interest rate of R^I. A key friction in the model is interbank lending costs in the form of a strictly increasing and convex real cost, $f(I)$, for the lending bank, which represents interbank monitoring costs.

At $t = 2$, returns on remaining assets are paid, firms sell their output to households at a price of P per unit, and households consume the goods they purchase. Deposits made at $t = 0$ and not withdrawn at $t = 1$ yield a return of R^{D0} at date 2, while deposits that are made at $t = 1$ yield a return of R^{D1} at date 2. Similarly, equity issued at date t yields a return of R^{Et} at date 2, $t = 1, 2$. MMF shares purchased at date t offer a competitive return of R^{Ft} at date 2, $t = 0, 1$. Thus, we consider shares of the MMF offered in different periods as investments in different funds. Finally, households pay a lump-sum tax (τ) such that the government maintains a net balanced budget.

Households obtain a nonpecuniary real liquidity benefit $\sigma > 0$ for holding a liquid asset. In our model, all assets held by households are liquid except equity. Thus, the total nominal utility benefit to holding a liquid asset with pecuniary nominal return $\mathbf{R}$ is $\mathbf{R}+\theta$, where $\theta = P\sigma$. This implies that equity is socially costly, a second key friction in the model.

The final key friction is bank moral hazard in the form of inefficient risk-shifting. Banks have the ability to shift risk at the end of dates $t = 0$ and $t = 1$. If a bank risk-shifts at date $t = 0$, it obtains an additional $\alpha(A^0)A^0$ in profits at the end of $t = 2$ with probability $\frac{1}{2}$, where $A^0 \equiv L + M^0$ is defined as the banks total assets, and where $\alpha(.) \geq 0$ is a weakly increasing and weakly convex function that allows the bank to risk-shift their assets in an amount at the margin that increases with the bank's balance sheet size. Since bank assets equal bank liabilities, $L+M^0 = D^0+E^0$. Alternatively, with probability $\frac{1}{2}$, the bank loses $\beta(A^0)A^0$ in profits, where $\beta(.)$ is a weakly increasing and weakly convex function. Similarly, the bank can risk-shift at the end of $t = 1$ on new assets acquired at $t = 1$ to obtain an additional $\alpha(A^0+A^1)A^1$ or lose $\beta(A^0 + A^1)A^1$, each with probability $\frac{1}{2}$, where A^1 are new bank assets (hence equal to new bank liabilities). Note that only the bank in the non-shocked sector can risk-shift at $t = 1$, since it is the only bank with new assets at that date. Risk-shifting that results in gains is observable but not verifiable nor contractible.

The loss function $\beta(.)$ is large enough such that, for any asset size, all bank profits and equity returns are zero, and with limited bank liability, the bank incurs a partial default on depositors. If the bank chooses large enough equity, it will not risk-shift, as the bank operates to ensure suitable returns for the equity shareholders. We assume that the government cannot commit not to bailout depositors when there is a default. Risk-shifting would impose the cost of bailouts on the government. Assuming $\beta(.)$ sufficiently larger than $\alpha(.)$, bank risk-shifting is socially inefficient. A bank regulator would require an external leverage requirement in place such that banks hold enough equity that risk-shifting is not incentive compatible for banks. This leverage ratio acts as a "balance-sheet cost", since it is costly for banks to increase the size of their balance sheet.[31] Indeed, the leverage ratio is, in itself, a source of economic inefficiency since households derive a non-pecuniary benefit from holding debt rather than equity. Thus, regulators will require that banks hold only the minimal amount of equity so that risk-shifting will be suboptimal in both periods for both banks.

Denote Π^B as the bank's profits in the absence of risk shifting and $\Pi_t^{B,RS}$ as the bank's profits under risk shifting at time t (given risk shifting is sucessful). We will have

$$\begin{aligned}\Pi_0^{B,RS} &\equiv \Pi^B + \alpha(A^0)A^0 - E^0 R^{E0} \\ \Pi_1^{B,RS} &\equiv \Pi^B + \alpha(A^0 + A^1)A^1 - E^1 R^{E1}\end{aligned} \tag{1}$$

and where the additional equity terms, $-E^0R^{E0}$ and $-E^1R^{E1}$, are paid by the bank to equityholders, in the outcome that the $a(.)$ is received, so that equity holders are fairly compensated under bank risk-shifting and are indifferent.

The constraint for banks to not risk-shift in $t = 0$ and $t = 1$ is

$$\Pi_t^{B,RS} \leq \Pi^B \text{ for } t = 0, 1$$

Note that we could add a third condition stating that banks should not want to risk-shift in both periods togeather, but that would be redundant as fulfillment of the two conditions in 1 guarantees fulfillment of the third. Two sufficient conditions for the constraint to hold is that

$$\begin{aligned}E^0 &= \frac{\alpha(A^0)A^0}{R^{E0}} \\ E^1 &= \frac{\alpha(A^0 + A^1)A^1}{R^{E1}},\end{aligned}$$

While these need not be necessary conditions, as will be shown later, these are the equity levels eliminate risk-shifting with minimal welfare costs. Thus, we assume the regulator imposes these leverage requirements on banks. The regulator can alter

[31] Bank balance sheet costs were introduced in MMS, in which each bank bears an exogenous cost that is increasing in the size of their balance sheet.

requirements in the face of changes of equilibrium parameters (e.g. R^{E0} and R^{E1}). However, banks take this requirement function as exogeneous and unchanging.

Frictions related to bank balance sheet size are motivated in part by the analysis of market observers. For example, interbank broker Wrightson ICAP (2008) voiced concerns that large reserves could "clog up bank balance sheets." In July 2013, the Federal Reserve and the FDIC proposed a new rule to strengthen leverage ratios for the largest, most systemically important banks. Under the proposed rule, bank holding companies with more than \$700 billion in consolidated total assets would be required to maintain a tier 1 capital leverage of 5 percent, 2 percent above the minimum supplementary leverage ratio of 3 percent. Such proposals suggest that exogenous regulatory-based balance sheet costs may be relevant in the near future in addition to the market-discipline based endogenous balance sheet costs that we derive.[32] An additional explanation for balance sheets costs in addition to capital requirements and leverage ratios as in the model is the FDIC deposit insurance assessment that is applied to all non-equity liabilities.

Furthermore, as MMS explains, banks tended to reduce the size of their balance sheets during the recent crisis, in line with the presence of balance sheet costs. Evidence for this cost is also suggested by figure 1. We observe that the quantity of reserves is clearly negatively correlated with all of the deposit and related short-term money market rates plotted. The table below lists these correlation coefficients.[33]

Rate	Correlation
Federal Funds Effective	-.59
O/N Eurodollar	-.57
4 Week T-Bill	-.53

In MMS, this negative correlation is explained by balance sheet frictions bearing exogenous costs on banks, which in equilibrium are pushed onto depositors. Thus, when reserves, and consequently bank balance sheets, are large, the resulting frictions are imposed on depositors through a lower deposit rate. [34]

[32] Source:http://www.federalreserve.gov/newsevents/press/bcreg/20130709a.htm

[33] Source: http://www.federalreserve.gov/releases/h15/update/ provides the Federal Reserve Board H.15 report and http://www.federalreserve.gov/Releases/h41/ provides the Federal Reserve Board H.4.1 report.

[34] Federal Reserve Bank of New York President Dudley states that "to the extent that the banks worry about their overall leverage ratios, it is possible that a large increase in excess reserves could conceivably diminish the willingness of banks to lend." Source: http://www.newyorkfed.org/newsevents/speeches/2009/dud090729.html

3.3 Optimizations

In this section we describe each agent's optimization. A bank's optimization is given by:

$$\begin{aligned}
\max_{L,M^0 D^0,D^1,E^0,E^1,I} \Pi^B \quad = \quad & \frac{1}{2}\{2R^L L - 2R^{E0}E^0 + R^{M^2}M^0 - R^{D0}D^0 \\
& + R^M(D^1 + E^1) - R^{D1}D^1 - R^{E1}E^1 + (R^I - R^M)I \\
& - P\int_0^I f(\hat{I})d\hat{I} - R^I \max\{0, \lambda(D^0 + E^0)R^W - R^M M^0\} \\
& + R^M \max\{0, R^M M^0 - \lambda(D^0 + E^0)R^W\} \\
& - R^{D0}[(1-\lambda)(D^0 + E^0) - E^0]\} \\
& s.t. \qquad L + M^0 = D^0 + E^0 \\
& \qquad\qquad R^M M^0 + D^1 - I \geq 0 \\
& \qquad\qquad E^0 = \frac{\alpha(A^0)A^0}{R^{E0}},\ E^1 = \frac{\alpha(A^0 + A^1)A^1}{R^{E1}}
\end{aligned}$$

The bank receives a certain return of $R^L L$ on its loans and pays $R^{E0}E^0$ in all states. If the bank is not in the shocked sector they obtain $R^{M^2}(M^0 - L)$ on their $t = 0$ reserve holdings and pay out $R^{D0}D^0$ on all $t = 0$ deposits. It also receives new deposits and equity holdings which are invested in reserves at $t = 1$ and paid back at $t = 2$, which is captured by the quantity $R^M(D^1 + E^1) - R^{D1}D^1 - R^{E1}E^1$. The bank has the opportunity to make interbank loans which have a return of $(R^I - R^M)I - P\int_0^I f(\hat{I})d\hat{I}$. If the bank is in the shocked sector, it will pay $R^{D0}[R^W(1-\lambda)(D^0 + E^0) - E^0]$ for non-withdrawn deposits, and either earn $R^M \max\{0, R^M M^0 - R^W\lambda(D^0 + E^0)\}$ or pay $R^I \max\{0, \lambda(D^0 + E^0) - R^M M^0\}$, depending on whether M^0 is large enough to cover withdrawals. The first two constraints are simple budget balance constraints for $t = 1$ and $t = 2$. The last constraints are the no risk-shifting constraints.

Firms seek to maximize profits obtained from sales of real goods in $t = 2$. The firm in sector i solves:

$$\max_L \Pi^F = P\int_0^L r(\hat{L})d\hat{L} - R^L L$$

The MMF maximizes profits and solves:

$$\begin{aligned}
& \max_{B^H,F^0} R^B B^H - R^{F0}F^0 \\
& s.t.\ \ B^H = F^0
\end{aligned}$$

The MMF simply arbitrages, in the bond market, the funds obtained from selling their shares to households. They maximize the spread between the total bond return and the claims that they pay out to shares in $t = 2$.

The household of sector i solves:

$$
\begin{aligned}
\max_{D^0,E^0,F^0,B^1,D^1,E^1} \quad & \frac{1}{2P}\{2(R^{F0}+\theta)(F^0)+(R^{D0}+\theta)D^0+2R^{E0}E^0+R^{E1}E^1 \\
& +(R^{D1}+\theta)(B^1P^B-E^1)+(R^{D0}+\theta)[(D^0-\lambda(D^0+E^0)] \\
& +(R^{F0}+\theta)\left[\frac{\lambda(D^0+E^0)R^W}{P^B}\right]-(R^{F0}+\theta)B^1-2\tau \\
& +\Pi^B+\Pi^F\} \\
& s.t. \qquad D^0+E^0+F^0<W \\
& \qquad\qquad B^1<F^0
\end{aligned}
$$

Households value real consumption, thus nominal returns are divided by the price level. MMF shares issue a riskless return of $R^{F0}+\theta$, which captures the first term in the objective function. With $\frac{1}{2}$ probability, the household is not shocked and earns $(R^{D0}+\theta)D^0+2R^{E0}E^0$ on $t=0$ assets. They also can sell B^1 of their MMF shares and invest them in equity or deposits to obtain $(R^{D1}+\theta)(B^1P^B-E^1)+R^{E1}E^1-(R^{F0}+\theta)B^1$. With $\frac{1}{2}$ probability, the household is hit with the shock and earns $(R^{D0}+\theta)[D^0-\lambda(D^0+E^0]$ on non-withdrawn deposits. They earn $(R^{F0}+\theta)\left[\frac{\lambda(D^0+E^0)R^W}{P^B}\right]$ on deposits withdrawn to purchase MMF shares. Finally, they pay τ in all cases and receive residual claims on the banks and firms, $\Pi^B+\Pi^F$. The first constraint assures household budget balance at $t=0$, while the second mandates that MMF share sales at $t=1$ do not exceed the household's holding of MMF shares.

3.4 Equilibrium Analysis

We use general equilibrium as our solution concept. In particular, an equilibrium in this economy is a set returns, $R^{D0}, R^{D1}, R^{E0}, R^{E1}, R^B, R^{F0}, P^B, R^L$, and R^I, and a $t=2$ price level P, such that all markets clear at the agents' optimizing levels of investment and consumption.

We assume standard regularity conditions:

$$
\begin{aligned}
r(L) &> 0,\ r'(L)<0,\ r(0)=\infty,\ r(\infty)=1 \\
f(I) &> 0,\ f'(I)>0,\ f(0)=0,\ f(\infty)=\infty \\
\alpha(D) &\geq 0,\ \alpha'(D)\geq 0,\ \alpha(0)=0,\ \alpha(\infty)=\infty.
\end{aligned}
$$

Since asset holdings are ex-ante identical, $M^0 = \frac{M}{2}$ and $F^0 = \frac{B-M}{2}$. This implies that

$$\begin{aligned} A^0 &= W - \frac{B-M}{2},\; A^1 = \lambda A^0 R^W \qquad (2) \\ L &= W - \frac{B}{2}. \qquad (3) \end{aligned}$$

We can now turn to the determination of equilibrium rates and quantities. We first discuss equity. By (2), A^0 and A^1 must both be positive in equilibrium. Thus, we must have both E^0 and E^1 positive in equilibrium to achieve no risk-shifting for $\alpha'(.) > 0$. Because deposits have a liquidity premium for households over equity, we have that $R^{E0} = R^{D0} + \theta$ and $R^{E1} = R^{D1} + \theta$. Since deposits are a cheaper form of capital, it is clear the the risk-shifiting constraint binds and no excess equity will be held, i.e.:

$$\begin{aligned} E^0 &= \frac{\alpha(A^0)A^0}{R^{E0}} \\ E^1 &= \frac{\alpha(A^0 + A^1)A^1}{R^{E1}} \end{aligned}$$

Before we state the main proposition of this section, our model necessitates several regularity conditions. First, we make the following assumption:

$$R^{M^2} > R^M + \max(\frac{\theta}{2}(2-\lambda), \alpha'(W)W + \alpha(W)) \qquad (4)$$

This assumes that R^{M^2} is sufficiently larger than R^M and will be necessary to allow equilibria without unnecessary withdrawals at $t = 0$. Second, we will assume that $R^M > \lambda R^W$, so that increases in reserves weakly decrease interbank lending. Third, we will assume that

$$\begin{aligned} R^M - \theta &> 0 \\ R^M - (\alpha'(A^0 + A^1)A^1 + \alpha(A^0 + A^1)) &> 0 \end{aligned}$$

This will ensure that equilibrium solutions for R^{D1} exist and remain positive. Third, we define

$$
\begin{aligned}
\gamma(x,y,z) &\equiv -[2R^{M^2} + (R^M - \lambda R^W) f(y - R^M z)\frac{2R^{M^2}}{2r(E - \frac{B}{2}) - R^M f(y - R^M z)} \\
&\quad - \frac{\theta\alpha'(x+y)y}{\frac{1}{2}(R^M - \theta) + \frac{1}{2}\{(R^M + \theta)^2 + 4\theta[R^M - (\alpha'(x+y)y + \alpha(x+y))]\}^{\frac{1}{2}} + \theta} \\
&\quad -\lambda R^W R^M - (2-\lambda)\theta] \\
\varphi(x,y,z) &\equiv -\theta\{2R^{M^2} + (R^M - \lambda R^W) f(y - R^M z)\theta\frac{2R^{M^2}}{2r(E - \frac{B}{2}) - R^M f(y - R^M z)} \\
&\quad - \frac{\theta\alpha'(x+y)y}{\frac{1}{2}(R^M - \theta) + \frac{1}{2}\{(R^M + \theta)^2 + 4\theta[R^M - (\alpha'(x+y)y + \alpha(x+y))]\}^{\frac{1}{2}} + \theta} \\
&\quad -\lambda R^W R^M - 2[\alpha'(x)x + \alpha(x)]\}
\end{aligned}
$$

and assume that both $\gamma(W, \lambda R^W W, \frac{\lambda R^W W}{R^M})$, $\varphi(W, \lambda R^W W, \frac{\lambda R^W W}{R^M}) > 0$. This will ensure that equilibrium solutions for R^{D0} exist and remain positive.

The following proposition establishes the remainder of the equilibrium.

Proposition 1 *A unique competitive equilibrium is given by:*

1. $R^L = R^{M^2} + \frac{P}{2} R^M f(I)$
2. $R^{D0} = \frac{\gamma(A^0, A^1, M^0) + \sqrt{\gamma(A^0, A^1, M^0)^2 - 4(2-\lambda)\varphi(A^0, A^1, M^0)}}{2(2-\lambda)}$
3. $R^{D1} = \frac{1}{2}(R^M - \theta + [(R^M + \theta)^2 + 4\theta[R^M - (\alpha'(A^0 + A^1)A^1 + \alpha(A^0 + A^1))]^{\frac{1}{2}})$
4. $P^B = \frac{R^B + \theta}{R^{D1} + \theta}$
5. $R^{F0} = R^B = R^{D0}(1 - \frac{\lambda}{2}) + \frac{\lambda}{2}[R^W R^{D1} - (R^W - 1)\theta]$
6. $R^W \leq \frac{R^{D0} + \theta}{R^{D1} + \theta}$
7. $I = \max\{0, \lambda A^0 R^W - R^M M^0\}$
8. $R^I = R^M + P f(I)$
9. $P = \frac{2R^{M^2}}{2r(L) - R^M f(I)}$.

Item 1 states that when $I > 0$, loan rates are at a spread above R^{M^2} increasing in I. Items 2 and 3 describe the equilibrium deposit rates. Item 4 shows that the equilibrium spot price of $t = 0$ MMF shares in $t = 1$ is given by the ratio of the total nominal utility value of bonds to $t = 1$ deposits. Item 5 describes the equilibrium bond rate, while item 6 places an upper bound on R^W. Item 7 shows that interbank loans are zero when they are not needed to fund shocks and are equal to the deficiency otherwise. The interbank loan rate, given in item 8, is above R^M at a spread also increasing in the amount of interbank loans, reflecting the real interbank monitoring cost $f(I)$.

Note at a bank's balance sheet size will increase with increases in reserves. We define bank balance sheet costs as

$$
\begin{aligned}
C_0(M^0) &\equiv R^{M^2} - R^{D0} && (5)\\
C_1(M^0) &\equiv R^M - R^{D1}. && (6)
\end{aligned}
$$

The following corollary to the proposition states that balance sheet costs increase, reflected in deposit rates R^{D0} and R^{D1} decreasing, with increases in reserves.

Corollary 2 $\frac{\partial C_0}{\partial M^0} < 0$, $\frac{\partial C_1}{\partial M^0} < 0$

Equity requirements act as a form of balance sheet costs for banks. Taking on more liabilities forces banks to hold more equity which is an expensive form of finance. Thus, there is friction in balance sheet expansion. Figure 2 provides a graphical illustration of the $t = 0$ bond, bank deposit, and bank loan markets.

See figure 2

A useful case is when $R^W \lambda A^0 < R^M M^0$, so that withdrawals can be funded without interbank trading. The level of reserves required for this is given by:

$$\overline{M} \equiv \frac{2\lambda R^W W - B}{R^M - \frac{\lambda}{2} R^W} \tag{7}$$

When $M \geq \overline{M}$, we have $I = 0$ and $R^L = R^{M^2}$.

3.5 Economic Welfare and Optimal Policy

Our model highlights two drivers of welfare: interbank monitoring costs and balance sheet costs. There is potentially a trade-off between these two costs. Increasing the supply of reserves reduces the need for interbank transactions and, thus, the cost associated with interbank monitoring. If reserves are large enough, however, the increase in the size of a bank's balance sheet raises equity requirements, which is costly.

In this section we consider a social planner, who chooses the optimal level of reserves, M^*, to maximize social welfare. Welfare is defined as the real utility of households, which is equivalent to the total sum of the real consumption and liquidity benefit enjoyed by households, minus interbank lending costs. More formally, M^* can be written as the solution to:

$$\max_{M} \quad \int_0^L r(l)dl + \sigma(B - M) + [2 + \lambda(R^W - 1)]\sigma(W + \frac{M}{2} - \frac{B}{2}) \tag{8}$$

$$-\sigma(2E^0 + E^1) - \int_0^I f(i)di \tag{9}$$

The first term represents total consumption in the economy determined by real production. This term is independent of the level of reserves as in MMS. The second term $\sigma(B-M)$ is simply the liquidity benefit households enjoy resulting from holding MMF shares that were invested in government bonds. The term $(W + \frac{M}{2} - \frac{B}{2})\sigma(2 + \lambda(R^W - 1))$ captures the liquidity benefit derived from households investing in bank assets (deposits and equity). Households would gain $2\sigma(W + \frac{M}{2} - \frac{B}{2})$ from investing in bank assets at $t = 0$ if no one withdrew at $t = 1$. Because of withdrawals and consequent new deposits at $t = 1$, a net benefit of $\lambda(R^W - 1)(W + \frac{M}{2} - \frac{B}{2})$ is added to the total bank benefit. If $R^W > 1$, this net benefit is positive and withdrawals multiply the bank asset benefit. If $R^W < 1$, the withdrawals decrease the total benefit from holding bank assets. The final term is the interbank lending cost.

The term $\sigma(2E^0 + E^1)$ represents the fact that a portion of bank assets must be held in equity, which does not yield a liquidity benefit and therefore must be subtracted off the prior term. Notice that equity issued at time zero is socially more costly than equity issued at time one. This is because both banks must issue equity at $t = 0$ and only the non-shocked bank issues equity at $t = 1$. Thus, equity costs are minimized when banks issue only the exact amount of equity needed to eliminate risk-shifting at $t = 0$. In other words, it is socially inefficient for banks to issue extra equity issued at $t = 0$ and reduce the amount of equity they must hold at $t = 1$. With this result, the leverage requirement we imposed is the socially optimal mechanism for eliminating risk-shifting.

It should first be noted that if the social planner has the ability to alter R^M as well as M a solution to his problem does not exist. To see this, note that for any chosen level of M, If we increase R^M we will weakly decrease $\int_0^I f(i)di$ while simultaneously strictly decrease $2E^0 + E^1$. Thus, for any selection of M, R^M we can find a Pareto superior policy simply by increasing R^M. However, increases in R^M will be extremely inflationary. It is widely accepted that high inflation yields economic inefficiencies, these however are not captured in our model. For this reason, we do not consider R^M as a choice variable for the planner and instead take this as exogenous to his problem.

The first order condition for the planner's optimization can be given as:

$$\frac{\sigma\lambda}{2}(R^W - 1) + (R^M - \frac{\lambda}{2})f(I) = \sigma(2\frac{\partial E^0}{\partial M} + \frac{\partial E^1}{\partial M}) \tag{10}$$

If the balance sheet cost is positive for $M = \overline{M}$, then $M^* < \overline{M}$, since the monitoring cost goes to zero when loans go to zero. If the balance sheet cost is zero at $M = \overline{M}$, then $M^* > \overline{M}$. In particular, for $R^W \leq 1$, it is clear that $M^* < \overline{M}$ for $\alpha'(0) > 0$, since $2\frac{\partial E^0}{\partial M} + \frac{\partial E^1}{\partial M} > 0$. Thus, some interbank lending is desirable. It is also clear that, as long as equity requirements are not too fast increasing when $M = \overline{M}$, $M^* > \overline{M}$, and some reserves are also desirable to mitigate monitoring costs. For $R^W > 1$ and $\alpha'(0) > 0$, $M^* \geq \overline{M}$ depending on how high R^W is. But given item 6 in proposition 1, we shouldn't expect R^W to be excessively high.

This result suggests that public money in the form of reserves is desirable for economic welfare. Reserves are liquid and reliable claims to future wealth which can be used to finance liquidity shocks, and thus are valuable to banks as well as households. In an optimal solution, the central bank may decide to provide banks public money in the form of reserves, instead of allocating it all to households in the form of government bonds. At the same time, outstanding reserves increase banks' balance sheet size, which exacerbates banks' incentives to risk-shift. Thus, quantities of reserves need to be moderated. At the end of the next section, we discuss how a social planner can mitigate the latter cost of a large size of the central bank's balance sheet when additional central bank public money tools are available to provide for different compositions of the central bank's liabilities.

4 Central Bank Tools

We now use this framework to analyze the RRP and TDF. We assume that the MMF can invest in RRPs in addition to government bonds.[35] In contrast, only banks can invest in the TDF. The TDF serves as a substitute to reserves for banks, but does not bear the liquidity benefit to banks that reserves do. Both RRPs and the TDF substitute for reserves on the balance sheet of the central bank one for one. Thus, these new central bank liability facilities essentially "absorb" traditional central bank reserve liabilities. In this section we assume that we are in the no interbank

[35] Banks would not invest in the RRP at equilibrium rates below IOER, as we consider.

lending case, i.e. $M \geq \overline{M}$, so we need not consider whether or not reserves are large enough to support necessary interbank lending.

4.1 Overnight RRPs: Fixed-Quantity vs. Fixed-Rate

The purpose of an overnight RRP is to offer a short-term investment that is available whenever needed. The ON RRP is offered at $t = 1$ and allows the MMF to purchase additional assets. This, in turn, allows households selling bonds to purchase MMF shares as an alternative to redepositing in their bank. We do not consider ON RRPs at date 0 to focus instead on the role of ON RRP in mitigating the liquidity shock. In this section, we also assume that we are in the no interbank lending case, i.e. $M \geq \overline{M}$, so we need not consider whether or not reserves are large enough to support necessary interbank lending.

Proposition 4 considers the case of a fixed-quantity operation (ON FQ RRP). The central bank offers a perfectly inelastic supply of RRPs (RP^{FQ}) at a market equilibrium competitive rate (R^{FQ}).

Proposition 3 *For $RP^{FQ} \leq \lambda(D^0 + E^0)R^W$, we have in equilibrium that $R^{FQ} = R^{D1}$ and $D^1 = \lambda(D^0 + E^0)R^W - RP^{FQ}$. Furthermore, in this equilibrium R^{D0}, R^{D1}, and R^B are all higher than the corresponding rates in proposition 1.*

Proposition 5 shows that a fixed-rate, full-allotment ON RRP (ON FRFA RRP) can achieve the same allocation as an ON FQ RRP. ON FRFA RRP offers an interest rate R^{FR} at $t = 1$ for any quantity demanded. The rate R^{FR} is set exogenously by the central bank.

Proposition 4 *If the central banks sets $R^{FR} = R^{FQ}$ from proposition 4, we have that $RP^{FR} = RP^{FQ}$, $R^{FR} = R^{FQ} = R^{D1}$, and $D^1 = \lambda(D^0 + E^0)R^W - RP^{FQ} = \lambda(D^0 + E^0)R^W - RP^{FR}$.*

Figure 5 illustrates the relationship between the two overnight RRP policies.

See figure 3

We can see from the previous two propositions that the RRP creates a transfer from the non-shocked bank to the shocked household by absorbing some of the liquidity shock. As a result, the equilibrium $t = 1$ deposit rate is increased, up to the facility rate, and the quantity decreased, by the size of the facility. The overall result of this is a decrease in bank profits (because a lower amount of $t = 1$ deposits are issued at a higher rate) and an increase in direct returns to households through higher returns on MMF shares.

Note that the presence of the ON RRP indirectly exerts upward pressure on bond rates in $t = 0$. The increase in the $t = 1$ deposit rate for a shocked household increases the overall expected return of investing in deposits. Arbitrage then require the bond and $t = 0$ MMF return to increase as well.

4.1.1 Uncertainty in Shock Size

While proposition 5 shows that a ON FQ RRP can implement the same allocation as an ON FRFA RRP, we consider how the two tools differ in a richer setting. In particular, the two facilities would have different implications if the fraction of household that are relocated, λ, is uncertain. In such a case, an ON FQ RRP would result in fluctuations in the RRP rate, while an ON FRFA RRP would result in fluctuations in the quantity of RRPs. Hence, a policymaker who dislikes fluctuations in the interest rate more than fluctuations in the quantity of reserves would prefer the ON FRFA RRP.

To formalize this, we assume in this section that λ is random and can take two realizations, λ^L and λ^H, with $\lambda^H > \lambda^L$. We assume that the central bank knows the two possible values of λ but does not know which one will occur when they implement their ON RRP policy.[36] We also assume that the central bank would like to target a specific $t = 1$ investment rate of R^{D1*} and can choose either an ON FRFA RRP or an ON FQ RRP to do so. We will show that, in general, the ON FRFA RRP can implement a $t = 1$ investment rate equal to R^{D1*} with less interest rate volatility than the ON FQ RRP.

To make the problem interesting, we analyze the case where:

$$\begin{aligned} & \frac{1}{2}(R^M - \theta) + \frac{1}{2}\{(R^M + \theta)^2 - 4[\alpha'(A^0)A^1 + \alpha(A^0)]\}^{\frac{1}{2}} \\ > \ & R^{D1*} \\ > \ & \frac{1}{2}(R^M - \theta) + \frac{1}{2}\{(R^M + \theta)^2 - 4[\alpha'(A^0 + \lambda^L A^0 R^W)A^1 + \alpha(A^0 + \lambda^L A^0 R^W)]\}^{\frac{1}{2}} \end{aligned} \tag{11}$$

so that the target $t = 1$ investment rate is higher than the outcome that would occur in either state without central bank intervention, but not so high that $t = 1$ deposit markets become completely inactive in all situations. We first show that an ON FRFA RRP policy can implement R^{D1*} in either state.

[36] For the analysis we conduct here, it is actually not necessary for us to define probabilities of the two states occuring.

Proposition 5 *If the central bank sets $R^{FR} = R^{D1*}$, and (11) holds, then $R^{D1} = R^{FR} = R^{D1*}$ when either λ^L or λ^H occurs.*

The proposition establishes that an ON FRFA RRP can impose a target R^{D1*} for any state under (11) and thus completely eliminate $t = 1$ interest rate volatility. It is rather clear that this cannot be achieved with the ON FQ RRP. This is because when λ^L is realized, a smaller quantity facility will be needed to impose R^{D1*} than when λ^H occurs. A central bank cannot achieve R^{D1*} for all realizations of λ if it cannot condition its policy on the state of the world. We do however show that the central bank can implement a floor on rates at R^{D1*}. First, we define RP^{FQ^*} to be the quantity of the facility that is needed to impose R^{D1*} when λ^H occurs; i.e. RP^{FQ^*} will solve:

$$
\begin{aligned}
&R^{D1*} = \frac{1}{2}(R^M - \theta) \\
&+\frac{1}{2}\{(R^M + \theta)^2 - 4[\alpha'(A^0 + \lambda^H A^0 R^W - RP^{FQ^*})A^1 + \alpha(A^0 + \lambda^H A^0 R^W - RP^{FQ^*})]\}^{\frac{1}{2}}
\end{aligned}
$$

Such an RP^{FQ^*} will exist by continuity as well as (11).

Proposition 6 *Under (11), if the central banks sets $RP^{FQ} = RP^{FQ^*}$, then $R^{D1} = R^{D1*}$ when λ^H occurs, and $R^{D1} > R^{D1*}$ when λ^L occurs.*

This proposition shows that a ON FQ RRP can effectively provide a floor on rates at the target rate R^{D1*}. However, $t = 1$ investment rates may be very volatile, especially if the difference between the two shock sizes are large and both occur with high probabilities. A central bank with the intention of implementing a target rate while minimizing interest rate volatility may thus prefer a ON FRFA RRP over an ON FQ RRP.

4.1.2 Discussion

Other advantages of a FRFA RRP may also exist that are outside the scope of this model. For example, fixed-rate RRPs provide MMFs with certainty regarding fixed-rates, and certainty regarding (unlimited) quantities, both of which would have additional benefits to MMFs in the face of uncertainty on demands and supplies in short-term money markets. In practice, MMFs have effective risk aversion, in part caused by the requirement for stable NAVs. The certainty provided by ON FRFA RRPs creates a benefit for a more stable transmission of monetary policy. FQ RRPs, in contrast, will not eliminate the uncertainty regarding equilibrium rates and quantities that MMFs can receive.

Furthermore, fixed-rate RRPs tend to better facilitate an overnight RRP facility. As shown above, such daily availability provides MMFs with greater certainty to support their elastic demand, at or above the RRP rate, for other assets. Fixed-rate RRPs also allow for one-day maturity RRPs, which can be rolled over. A fixed-quantity, operation-based RRP is not as amenable to daily operations. FQ RRPs would therefore tend to require longer term RRPs for operational cost reasons. Shorter-term RRPs provide MMFs the ability to substitute with shorter-term alternative assets. Such RRPs will support money market and bank deposit rates of all maturities, even as short as overnight rates. Typically, money market rates increase with the tenor of the instrument. While ON FRFA RRPs can be rolled-over, and therefore provide a better floor to overnight rates as well as to longer-term rates, a longer-tenor RRP does not provide such support to rates of shorter tenors.

As a result of this, longer-term RRPs may possibly have little effect on shorter-term rates. For example, a one-month RRP may increase one-month money market rates, but this may not be well transmitted down to provide support for overnight rates, which could be best supported with ON RRPs. One-month RRPs may rather simply increase the steepness of the one-month yield curve. In the case of ON FRFA RRPs, MMFs may even take up quantities of RRPs at quite low rates.

4.2 Term vs. Overnight RRPs

We model long-term RRPs as an alternative source of investment for MMFs in $t = 0$. Term RRPs (denoted RP^{TM}) are supplied inelastically by the central bank at a fixed-rate R^{TM}, paid off in $t = 2$. Term RRPs are available only to the MMF and reduce the quantity of reserves. We restrict our analysis to the case where the interbank market is inactive at date 1, as in proposition 1. If RP^{TM} RRPs are issued in $t = 0$, the total supply of reserves decreases from M to M' where $M' = M - RP^{TM}$, so we consider $M' \geq \overline{M}$.

Item 5 of proposition 1 must hold for bonds to be held in equilibrium. We define the equilibrium bond rate in the benchmark case of proposition 1 to be $\overline{R^B}$. If term RRPs are introduced at rate below $\overline{R^B}$, then no RRPs are held by the MMF. Therefore we have the following lemma:

Lemma 7 *If R^{TM} is set less than $\overline{R^B}$, we will have $R^{TM} < R^{F0} = R^B = \overline{R^B} = R^B = (1 - \frac{\lambda}{2})R^{D0} + \frac{\lambda}{2}(R^W R^{D1} - \theta(R^W - 1))$ and $RP^{TM} = 0$, $M = M'$.*

If R^{TM} is set above $\overline{R^B}$, then there will be a positive demand for term RRPs. The following proposition shows that the central bank has the freedom to set the RRP rate within an appropriate range without inducing interbank lending.

Proposition 8 *There exists a* $\widehat{R^{TM}}$ *such that for* $\overline{R^B} < R^{TM} < \widehat{R^{TM}}$ *, in equilibrium we must have* $\overline{R^B} < R^B = R^{TM} = R^B = (1-\frac{\lambda}{2})R^{D0} + \frac{\lambda}{2}(R^W R^{D1} - \theta(R^W - 1))$*,* $RP^{TM} > 0$*,* $M' < M$*, and* $M' \geq \overline{M}$*. Furthermore, both* R^{D0} *and* R^{D1} *will increase, while their respective equilibrium quantities decrease.*

RRP holdings increase when the term RRP rate is set at a higher level. This leads to higher MMF investment and lower bank liabilities, reducing equity requirements. Hence, both R^{D1} and R^{D0} increase until $R^{TM} = (1-\frac{\lambda}{2})R^{D0} + \frac{\lambda}{2}[R^W R^{D1} - \theta(R^W - 1)]$. The term RRP rate, when it is set sufficiently high, creates a floor for the bond and deposit rate. Figure 6 graphically illustrates the effect of the term RRP policy on R^{D0}, and the affect on R^{D1} is similar.

See figure 4

We have modeled term RRPs as having a fixed-price and full-allotment. As in the previous section, one can also consider an operation for a fixed quantity, RP^{TM}, of the RRP asset, where the rate is market determined. The equilibrium in the fixed-price, full-allotment case where $\overline{R^B} = R^B = R^{F0} = R^{TM}$ is identical to any equilibrium in which a quantity RP^{TM} of RRPs are set that yield R^{TM} in equilibrium. The main difference is that in the fixed-rate setting we could have equilibria where $R^{TM} < R^B$. In this situation, MMFs only hold bonds and no RRPs are held. Such an equilibrium (where the RRP rate is strictly below the other rates) is impossible in the operation style setting of the fixed-quantity RRP market.[37] Arbitrage forces the RRP rate to be equal to the bond rate, since it is also a safe asset. For similar reasons, the bank deposit rates increase with the RRP quantity supplied. The mechanism is that an increase in the quantity supplied of RRPs decreases the quantity of deposits which reduces balance sheet costs.

Both the term and the ON RRP increase $t = 1$ and $t = 2$ deposit rates. However, they do so through different mechanisms. The ON RRP (that is offered in $t = 1$ only) sets a reservation $t = 1$ deposit rate, which directly raises R^{D1} up to that level. R^{D0} increases because the ON RRP absorbs shocked withdrawals and lowers $t = 1$ expected balance sheet costs for the bank. The term RRP (offered only in $t = 0$) directly lowers balance sheet costs in $t = 0$ and partially raises $t = 0$ deposit rates. The decrease in deposits at $t = 0$ reduces the size of the shock in $t = 1$, which then indirectly increases $t = 1$ deposit rates by reducing the balance sheet cost burden. This, then, feeds back into $t = 0$ deposit rates, as banks now expect lower $t = 1$ deposits. Thus, an additional increase in R^{D0} occurs.

[37] One could argue that this case corresponds to a zero quantity auctioned, where the equilibrium rate i is indeterminate within a range.

An ON RRP (of either fixed-rate or fixed-quantity) offered at $t = 0$ would also reduce D^0 and thus reduce balance sheet costs and raise $t = 0$ deposit rates. Term RRPs may be preferable, however, if offering RRPs of open-ended size on a daily basis is operationally difficult or expensive compared to a term RRP that attracts large, long-term deposits.

4.3 TDFs vs. RRPs

As shown in the previous sections, RRPs provide a tool for the central bank to both manage reserves and provide floors for rates. Another such tool is the Term Deposit Facility (T). The TDF is comparable to the term RRP with the exception that they are only offered to banks. As we will show in this section, the two tools vary in their effect on equilibrium rates depending on various parameters in the model. For this section, we assume that the central bank chooses a sufficiently large quantity of the TDF or RRP such that reserves are reduced to the point where $M < \overline{M}$ at the end of $t = 0$. That is, the central bank is using its policy tools in large enough size tools so as to rekindle the interbank market and promote interbank lending in $t = 1$.

We model the TDF as a fixed-quantity operation (T) offered by the central bank in $t = 0$ maturing in $t = 2$ with a competitive equilibrium return R^T. Important to note is that TDF holdings cannot be used to ward off liquidity shocks. In this sense, they are a perfect substitute for real sector bank lending and in equilibrium we must have that $R^T = R^L$. The key difference between the RRP and the TDF is that the TDF forces substitution of liquid reserves to illiquid TDF holdings completely within the banking sector. RRPs, on the other hand, divert reserve holdings to the assets held by the universal MMF which is not prone to shocks. Thus, utilizing RRPs as opposed to TDFs reduces liquid assets while bearing less of an increase on interbank borrowing costs and liquidity premia. For simplicity, and for ease of comparison, we will assume that the RRP is a term RRP in this case.

To see this, first suppose the central bank increases the TDF supply by 2 units. Each bank would decrease its liquid reserve holdings by 1 unit. In doing so, they lose R^M units of reserves that could have been used to ward off a potential liquidity shock. This means that interbank loans will increase by R^M, which implies that the equilibrium interbank rate will increase by $f(I + R^M) - f'(I)$. On the other hand, if the central bank were to supply 2 units more RRPs instead, each bank would still decrease its liquid reserve holdings by 1 unit (assuming the MMF borrows equally from both sectors), sacrificing R^M in liquid assets in $t = 1$. However, each household is also converting one deposit to an MMF share. Thus, interbank loans will only increase by $R^M - \lambda < R^M$ and the interbank rate will increase by $f(I + R^M - \lambda R^W) - f'(I) < f(I + R^M) - f'(I)$. This implies that the liquidity premium will be smaller in this case than the former. This is shown in figure 7.

See figure 5

This difference also factors into the effect on deposit rates. From part 2 of proposition 1, we can see that the $t = 0$ deposit rate is increasing in interbank lending and decreasing in equilibrium $t = 0$ and $t = 1$ liabilities due to equity requirements. This leads to the following proposition for interbank costs and equity requirements that are largely convex.

Proposition 9 *When marginal equity requirements are large relative to marginal interbank lending costs and both are convex, we will have* $\frac{\partial R^{D0}}{\partial T} > \frac{\partial R^{D0}}{\partial RP^{TM}}$. *This occurs when liabilities and liquid reserves are relatively large and interbank lending is relatively small. Reversing these relationships will yield the opposite inequality.*

This result is immidiate due to the convexity of the two costs. Nevertheless, there are relatively interesting implications of the above proposition. The sizes of $f(.)$ and $c(.)$ will increase when interbank loans and deposits, respectively, are larger. Therefore, when both of the cost functions are very convex, we suspect that a policymaker who has a primary goal of increasing deposit rates may want to mediate between usage of both tools. Specifically, he may want to first use term RRPs to reduce deposit size, then divert to implementation of the TDF after decreases in marginal balance sheet costs diminish. On the other hand, a policymaker who seeks to absorb reserves with a smaller effect on increasing deposit rates may want to focus on one facility, namely, the TDF if marginal balance sheet costs are very fast increasing, and the RRP if interbank lending frictions are instead more prominent.

4.4 Optimal Monetary Policy with An Extended Set of Tools

In this section, we extend the analysis performed at the end of section 3, but allow the planner access to RRPs and the TDF in addition to reserves. The same trade-off remains at the heart of the planner's problem. Liquidity helps banks reduce monitoring costs but comes at a cost of larger equity requirements.

The first thing that is immediately clear is that the TDF provides no welfare benefits. The TDF increases interbank lending without reducing equity requirements, as it keeps bank's asset size constant. In any economic welfare maximizing monetary policy, the size of the TDF should be zero.

The ON RRP, however, should be used to its maximum potential. Reserves at $t = 0$ are necessary buffers against liquidity shocks. However, adding reserves to the banking system beyond that point can be wasteful in terms of extra equity requirements. Investment is better made into the ON RRP. Thus the ON RRP should clearly be set to completely eliminate the shock. In particular, for whatever bank balance sheet size, A^0, the planner chooses as optimal, the size of the ON RRP should be set to completely cover the size $\lambda A^0 R^W$. If λ is uncertain, the size of the ON RRP should be set to eliminate the shock in the worst state (highest λ).

The planner's problem then becomes choosing the M^* that maximizes:

$$\max_M \int_0^L r(l)dl + \sigma(B-M) + (W + \frac{M}{2} - \frac{B}{2})\sigma[2 + \lambda(R^W - 1)] - \sigma(2E^0) - \int_0^I f(i)di \quad (12)$$

Note that (12) is identical to (8) except that now unnecessary $t = 1$ equity requirements are eliminated as withdrawn funds are entirely invested in overnight RRPs instead of bank assets. Thus, we will have that the M^* that maximizes (12) will be weakly greater than the M^* that maximizes (8). When equity requirements diminish, public money becomes more valuable to the economy. In particular, $M^* < \overline{M}$ when R^W is sufficiently small and $\alpha(.) > 0$, but still greater than zero as long as $t = 0$ equity requirements are not too fast increasing when $M = 0$.

Obtaining the M^* that maximizes (12) can either be done through standard bond sales or by utilizing term RRPs at $t = 0$. Our model does not distinguish between the two. However, if the central bank prefers to keep longer term bonds on their balance sheet, term RRPs may be the more desirable approach.

5 Extensions

In this section we consider two extensions introducing limited banking competition in the deposit market to the model. In the first extension, we assume banks are monopolistic suppliers of deposits within their sectors. In the second, the two banks across sectors engage in Bertrand competition in their deposit contracts.

The purpose of this section is twofold. First, it serves as an opportunity for us to discuss our tools in a slightly more elaborate framework. Second, and perhaps more importantly, it provides additional reasons as to why we observe money market rates significantly below IOER. In the competitive case, the spread was entirely due to pecuniary costs of balance sheet expansion that the bank had to be compensated for. When there is limited competition, banks gain strong opportunities to extract surplus from households by offering lower rates. Indeed, in the following two sections, banks may offer deposit contracts with even lower returns then in the competitive case.

In the monopolistic case, we find that banks naturally offer the lowest rate at which households would be willing to hold deposits. This indifference rate can be raised by usage of the tools. In the Bertrand case, many contracts are possible in equilibrium. Some of these contracts can be well below the competitive level depending on parameters. The tools can also be used in this situation to eliminate those potential contracts in equilibrium.

A key assumption of these two sections that was not used in previous sections is the ability for households to store wealth. This assumption is not necessary for these sections either, but are more realistic and make the problem significantly more interesting by introducing a strong individual rationality constraint for households to be willing to hold financial assets. In particular, in the following two sections, we assume households can store their endowment at $t = 0$ until $t = 2$ for a real return of $1 + \sigma$ (nominal return of $P + \theta$). At $t = 1$, we assume that households can store any cash from financial investments at $t = 0$ for a nominal return of $1 + \theta$. Should investment opportunities prove less profitable than storage, households will store their wealth instead of investing it.

5.1 Monopolistic banks

In this section we consider the bank of each sector being the monopolistic supplier of deposits to households. A key insight is that all quantities are pinned down in equilibrium by the economic constraints. Banks will therefore take these constraints as given and choose their deposit rates (R^{D0} and R^{D1}) and withdrawal rates (R^{W}) optimally. We also assume that banks have full knowledge of how other equilibrium returns and prices will evolve as a result of their actions.

First, if households have the ability to store cash, then it is clear that the optimal R^{D1} will be 1. Banks will give households exactly their outside option on deposit holdings at $t = 1$. Note that households of the non-shocked sector will still be willing to sell their bonds to the shocked sector as the price of bonds will rise in equilibrium: $P^{B} = \frac{R^{B}+\theta}{1+\theta}$. This occurs because shocked depositors have a perfectly inelastic demand for bonds at $t = 1$.

Moving backwards, banks will choose their optimal R^{D0} and R^{W}. Their optimization is essentially the same as in the benchmark model, however given that bank asset and liability quantities are pinned down in equilibrium, we can simplify the banks optimization to:

$$\max_{R^{D0},R^W} \frac{1}{2}\{-R^{D0}D^0 - (R^M - f(\lambda(D^0+E^0)R^W - R^M M^0))\max\{0, \lambda(D^0+E^0)R^W - R^M M^0\} \tag{13}$$

$$+ R^M \max\{0, R^M M^0 - \lambda A^0 R^W\} - R^{D0}[(1-\lambda)A^0 - E^0]$$

$$s.t. \qquad R^{D0} = \frac{P+\theta-\frac{\lambda}{2}[R^W(1-\theta)+\theta]}{1-\frac{\lambda}{2}}$$

The constraint is the household's individual rationality constraint that mandates that they are indifferent between storing their endowment or depositing in the bank.

First consider that the bank sets R^W low enough that interbank lending is not necessary in the event of the shock. Substituting the constraint in for R^W in the objective function we have that the derivative of the objective function with respect to R^W is given by:

$$\frac{\lambda(1-\theta)(D^0-E^0)}{2-\lambda} + \frac{\lambda(1-\theta)(1-\lambda)A^0}{2-\lambda} - A^0(\lambda R^M - \frac{\lambda(1-\lambda)(1-\theta)}{2-\lambda}) \tag{14}$$

With some algebra we can show that this derivative will be less than zero when:

$$A^0 > \frac{(1-\theta)(D^0-E^0)}{(2-\lambda)R^M - (1-\lambda)(1-\theta)} \tag{15}$$

Note that $A^0 > (1-\theta)(D^0-E^0)$ always and since $(1-\theta) < 1$ and $R^M > 1$, $(2-\lambda)R^M - (1-\lambda)(1-\theta) > 1$. Thus, (15) will always hold. This leads to the followning proposition

Proposition 10 *A monopolist will set* $R^W = 0$, *no interbank lending will occur, and* $R^{D0} = \frac{P+\theta(1-\frac{\lambda}{2})}{1-\frac{\lambda}{2}}$

The monopolist faces a trade off in that offering a lower deposit rate means that he must offer a higher withdrawal rate so that the expected return on a bank deposit is high enough to satisfy the households individual rationality constraint. The proposition states that the monopolistic bank would rather offer a higher $t = 0$ deposit rate than face the potential of any withdrawals at $t = 1$. Note that $R^{D0} = \frac{P+\theta(1-\frac{\lambda}{2})}{1-\frac{\lambda}{2}}$ is strictly greater than $P + \theta$ which is the households nominal return on a unit of storage. Also note that for smaller λ, R^{D0} will be smaller and closer to the households indifference rate $P + \theta$. If $P + \theta$ is significantly lower than the competitive deposit rate given in Proposition 1, then the monopolistic equilibrium can have money market returns well below IOER.

Tools can have interesting effects in this environment. First consider the ON RRP. The ON RRP will raise R^{D1} above 1 to the ON RRP rate. Thus, condition (15) will become

$$A^0 > \frac{(R^{D1} - \theta)(D^0 - E^0)}{(2-\lambda)R^M - (1-\lambda)(R^{D1} - \theta)} \tag{16}$$

For an ON RRP rate high enough, R^{D1} can be raised so high that condition (16) will no longer hold and the derivative of the bank's objective function with respect to the withdrawal rate will become positive. In such a situation, we will have that the bank may optimally choose a withdrawal rate high enough so that interbank lending will occur. Therefore, in a monopolistic setting, an aggressive ON RRP policy may in fact incentivize interbank lending. The intuition behind this is that increases in the ON RRP rate exogeneously increases the households return on a deposit. Thus the bank can offer much lower $t = 0$ deposit rates with small increases in the withdrawal rate. Similarly, a term RRP offered at $t = 0$ will reduce A^0 and also incentivize interbank lending at large enough sizes. Fixed rate RRP policies can clearly dramatically raise rates by increasing the profitability of the household's outside options. In fact, offering these facilities at a high enough rate can induce in the monopolist to again offer competitive rates, and no take up is necessary to induce these rate raises. If we are in a situation where (16) holds, then the TDF will have no effect. However, if the RRP has been used so that (16) is violated, the TDF can make interbank lending more costly and thus have a countervailing effect by decreasing R^W.

5.2 Bertrand competition

We now consider the two banks of the two sectors engaging in Bertrand competition, that is, households have the ability to choose between holding their deposits in either of the two sectors at $t = 0$.[38] One may think that this induces more competition than the competitive case. However, it is important to realize that while households are competitive across sectors, they hold full market power with their sector. In this situation, as we show below, there will be many equilibrium outcomes. As before we will have that $R^{D1} = 1$, as the non-shocked bank is the monopolistic supplier of $t = 1$ deposits. We aim to classify potential equilibrium R^{D0} and R^W.

Denote $\Pi^B(R^{D0}, R^W, A^0)$ as the bank's expected profit from offering households R^{D0} and R^W on a deposit contract and holding A^0 in assets at $t = 0$. We focus on

[38] For simplicity we maintain the assumption that only the non-shocked bank offers $t = 1$ deposits.

pure symmetric strategies and assume that the market is split evenly across sectors when households are indifferent between the contracts offered in the two sectors. That is, banks face the choice of either offering an equally attractive contract as their competitor and holding half the total equilibrium deposit amount across sectors ($E - \frac{B}{2} + \frac{M}{2}$) or offering a more attractive contract and attracting all deposits from both sectors ($E - B + M$). The latter option may be attractive because of increased revenue from investment, however banks also face increasing leverage requirements for maintaining larger balance sheets, so there is a trade off and balance sheet expansion may therefore not be optimal.

Let us define the set $\underline{\mathbb{C}}$ as follows:

$$\begin{aligned}\underline{\mathbb{C}} \;&=\; \{(R^{D0}, R^{W}) : \Pi^{B}(R^{D0}, R^{W}, E - \frac{B}{2} + \frac{M}{2}) \geq \Pi^{B}(R^{D0\prime}, R^{W\prime}, E - \frac{B}{2} + \frac{M}{2}),\ \forall (R^{D0\prime}, R^{W\prime}) \\ &\quad \text{s.t.}\ \ R^{D0}(1 - \frac{\lambda}{2}) + \frac{\lambda}{2}[R^{W} - (R^{W} - 1)\theta] = R^{D0\prime}(1 - \frac{\lambda}{2}) + \frac{\lambda}{2}[R^{W\prime} - (R^{W\prime} - 1)\theta]\}\end{aligned}$$

Elements of $\underline{\mathbb{C}}$ are optimal in the sense that they offer the the bank the highest expected profit out of all contracts that give households the same expected payoff when the deposit quantity is equal to half the market ($E - \frac{B}{2} + \frac{M}{2}$). If banks are comitted to offering households a certain expected payoff for $E - \frac{B}{2} + \frac{M}{2}$ deposits, the optimal contract will necessarily be an element of $\underline{\mathbb{C}}$.

Similarly define the set $\overline{\mathbb{C}}$ as:

$$\begin{aligned}\overline{\mathbb{C}} \;&=\; \{(R^{D0}, R^{W}) : \Pi^{B}(R^{D0}, R^{W}, E - B + M) \geq \Pi^{B}(R^{D0\prime}, R^{W\prime}, E - B + M),\ \forall (R^{D0\prime}, R^{W\prime}) \\ &\quad \text{s.t.}\ \ R^{D0}(1 - \frac{\lambda}{2}) + \frac{\lambda}{2}[R^{W} - (R^{W} - 1)\theta] = R^{D0\prime}(1 - \frac{\lambda}{2}) + \frac{\lambda}{2}[R^{W\prime} - (R^{W\prime} - 1)\theta]\}\end{aligned}$$

$\overline{\mathbb{C}}$ is the equivalent of $\underline{\mathbb{C}}$ with the deposit quantity being the whole market ($E - B + M$) as opposed to only half the market in $\underline{\mathbb{C}}$. If a bank wants to service the whole market while maintaining a certain level of household expected payoff, the optimal contract will be a member of $\overline{\mathbb{C}}$.

Finally define $\mathbb{C}^{*}$ as:

$$\begin{aligned}\mathbb{C}^{*} \;&=\; \{(R^{D0}, R^{W}) : (R^{D0}, R^{W}) \in \underline{\mathbb{C}} \text{ and} \\ &\quad \Pi^{B}(R^{D0}, R^{W}, E - \frac{B}{2} + \frac{M}{2}) \geq \Pi^{B}(R^{D0\prime}, R^{W\prime}, E - B + M),\ \forall (R^{D0\prime}, R^{W\prime}) \in \overline{\mathbb{C}} \\ &\quad \text{s.t.}\ \ R^{D0}(1 - \frac{\lambda}{2}) + \frac{\lambda}{2}[R^{W} - (R^{W} - 1)\theta] = R^{D0\prime}(1 - \frac{\lambda}{2}) + \frac{\lambda}{2}[R^{W\prime} - (R^{W\prime} - 1)\theta]\}\end{aligned}$$

The below proposition characterizes the space of potential equilibrium deposit contracts.

Proposition 11 *A necessary and sufficient condition for a pair* (R^{D0}, R^W) *to be an equilibrium contract is that* $(R^{D0}, R^W) \in \mathbb{C}^*$ *and* $R^{D0} \geq \frac{P+\theta-\frac{\lambda}{2}[R^W(1-\theta)+\theta]}{1-\frac{\lambda}{2}}$

$(R^{D0}, R^W) \in \mathbb{C}^*$ ensures that neither bank has the incentive to deviate to a contract that is either more profitable but equally preferred by the household, or to a contract that is slightly more preferred by the household and will yield the entire market. The second condition ensures households want to make positive deposits. Once again, general equilibrium and even market split necessitates that each bank will hold exactly $E - \frac{B}{2} + \frac{M}{2}$ in assets at $t = 0$ in any equilibrium.

It is clear that many contracts can satisfy the above proposition and it is difficult to characterize the space of admissable equlibrium contracts further than this. However, some additional intuition is perhaps necessary. Consider a potential equilibrium contract (R^{D0}, R^W). Since $(R^{D0}, R^W) \in \underline{\mathbb{C}}$ it is clear that no bank would want to deviate to a contract that yields only half the market. Consider a bank deciding whether to deviate to a contract $(R^{D0'}, R^{W\prime})$ that will yield them the entire market. In making this decision the bank faces a tradeoff. On the one hand, more assets allows the bank more investments and more raw revenue. However, a larger balance-sheet size also yeilds higher equity requirements. A contract (R^{D0}, R^W) can only be an equilibrium if the costs from balance-sheet expansion exeed the benefits. When R^{D0} is too low, the equity requirement costs will be small and the added revenue benefit of balance-sheet expansion will be large in comparison. However, a higher R^W is more costly for a large balance-sheet than a small one, so a low deposit rate can be accomodated with a higher withdrawal rate. So increasing the withdrawal rate in equilibrium relaxes both of the Bertrand oligopoly constraints. However, one thing that is clear in the Bertrand case is that the monopolistic equilibrium may not be feasible. The monopolist only needed to accomodate the second constraint of the above proposition, but a Bertrand equilibrium is also subject to the bank's incentive compatibility constraint that they should not want to capture the whole market. When the incentive compatibility constraint binds more strongly over the individual rationality constraint depends on parameters. In particular, when balance sheet costs are low or slowly increasing, there is more incentive to expand the balance sheet at low rates. Thus, if banks are subject to less profitable risk-shifting abilities, the incentive compatibility constraint may be tighter than the individual rationality constraint and rule out the monopolistic equilibrium in the Bertrand case.

While many equibria are possible, many equilibria can exist where large spreads exist between IOER and money market rates. We can also consider our tools in this situation. The ON RRP will work only through the households individual rationality constraint and will thus have the same effect of mandating higher withdrawal rates for a given deposit rate in equilibrium. The effect of the term RRP is somewhat ambiguous. It can expand the set of possible equilibria as it decreases the absolute size of the banking sector and thus the extra return to deviating and doubling asset holdings relaxing the incentive compatibility constraint (as well as the individual rationality constraint. However, it will also reduce the costs of doubling balance sheet size, thus increasing the incentives to deviate on any contract. In either case however, a fixed rate RRP will tighten the household's individual rationality constraint and raise the household's returns on permissible contracts. In the no interbank lending case, the TDF will have no effects on the contract space. However, in the positive interbank lending case, a large TDF increases the the costs interbank lending for bigger balance sheet sizes and can make deviation less attractive.

6 Conclusion

In response to the 2007-09 financial crisis and subsequent economic conditions, the Federal Reserve engaged in large-scale lending to financial institutions to provide liquidity and large-scale asset purchases to stimulate the economy by lowering interest rates. As a result of these policies, the amount of reserves in the banking system increased dramatically to over $2 trillion during 2013.

We introduce a simple general equilibrium model to analyze the tools available to the Federal Reserve to manage interest rates and the supply of reserves. Our model highlights numerous important results regarding IOER, RRPs, and the TDF. As in MMS, IOER represents the riskless return for holding marginal reserve on a bank's balance sheet and, thus, influences the deposit rate offered by banks. The IOER also sets a short-term reservation rate in the interbank market below which DIs will not lend. RRPs help reduce the balance sheet costs faced by banks and reduce the supply of reserves. RRPs also set a reservation interest rate for short-term deposits that occur as a result of liquidity shocks. While the model shows that fixed-rate and fixed-quantity ON RRPs would be identical in the absence of informational frictions, fixed-rate RRPs may be advantageous for a central banker trying to set a floor on short-term rates without perfect knowledge on the intensity of shocks or exogenous costs agents face. Term RRPs in this model offer little benefit over ON RRPs if the ON RRP can be offered on a consistent basis at low cost in implementation. However, they can further reduce balance sheet costs and raise deposit rates. TDFs, in contrast to RRPs, do not reduce bank equity requirements as they do not reduce bank balance sheet size. Therefore, when the banking system incurs costly monitoring on interbank markets, RRPs increase the size of this interbank market by less than the TDF. This motivates usage of the RRP and the TDF concurrently. Namely when marginal risk-shifting incentives and interbank lending costs are very convex, policy makers with the intention of raising deposit rates to high levels may want to use both facilities together.

In addition to our analysis of how interest rates and quantities are determined in equilibrium, we also analyze how the Federal Reserve can provide optimal levels of public money. In our model, public money refers to special assets that the Federal Reserve offers that are not available in the private economy alone. These include all of our tools, as well as reserves. Public money has benefits that show positive amounts of public money should be provided in an optimal equilibrium. Reserves serve as a liquid asset for banks that can be used to pay off withdrawals from depositors. Substantial reserves can prevent banks from having to excessively incur monitoring costs in interbank markets. However, too many reserves in the banking system exacerbates the size of banks' balance sheets with an increased moral hazard problem, which further increases equity that banks will hold as a commitment device not to risk-shift on assets. These equity requirements are socially costly when households place extra value on liquidity. Thus, RRPs are of use to absorb reserves from the banking system in excess of those used for mitigating liquidity shocks, which reduces the need for equity requirements. The Federal Reserve is able to provide these public assets as it is a reliable counterparty backed by government taxes not subject to equity requirements as commercial banks are.

We conclude our analysis with an extension to limited banking competition in the deposit market. We find that this may be an additional reason for spreads between IOER and money market rates. In this case, banks are able to extract more surplus from households by exerting market power. Monopolistic banks will offer a contract to households that makes them completely indifferent between storing wealth and holding deposits, which may yield household expected returns that are significantly below the competitive contract and IOER. When banks engage in Bertrand competition across sectors, many contracts are possible, but some of them can have equilibrium money market returns also below IOER and the competitive outcome. In both cases, the tools can be used to raise rates by providing households a stronger alternative option to the deposit market.

Our model provides a broad framework in which many additional questions about central bank policy for managing large levels of reserves can be analyzed. As in MMS, we can ask how balance sheet frictions, like more profitable risk-shifting on larger balance sheets, effect inflation and consumption. Additionally, we can consider heterogeneity across and within sectors. Other extensions to the model can include incorporating additional financial intermediaries, such as securities dealers and GSEs. The central bank can manage its liabilities facilities with regard to these financial institutions to further absorb reserves, influence the broad range of money market rates, and impact the level of lending, output, inflation, and consumption. The framework developed is also versatile to analyze a variety of further institutional details for financial intermediaries and money markets.

Appendix: Proofs

Proof of Proposition 1. Item 1 comes from the first order condition of the banks optimization with respect to loans. Item 4 comes from the first order condition of the banks optimization with respect to B^1. Item 5 comes from the first order condition of the household's optimization with respect to D^0. Item 6 is necessary to prevent unshocked deposits from being withdrawn and invested in bonds. If this were to occur no one would want to hold $t = 0$ deposits at $t = 1$. Everyone would withdraw their deposits at $t = 1$ to invest in bonds (since we do not allow redeposits in the same bank). However, the total bond supply at $t = 1$ would not allow this, so we must have item 6 hold in equilibrium. Item 8 comes from the first order condition of the banks optimization with respect to I. Given item 8, if $I > 0$ then $R^I > R^M$. Since R^M is the bank's best investment option, there is no reason to demand I unless it is necessary to fund withdrawals. Thus, $I = \lambda A^0 R^W - R^M M^0$ if this is positive and 0 otherwise. I is feasible as $\lambda R^W A^0$ is deposited into the lending bank. Item 9 comes from the first order condition of the firm's optimization with respect to L. Items 2 and 3 come from the first order conditions banks optimization with respect to D^0 and D^1 respectively, taking into consideration the equity requirements. For item 3, we will obtain

$$R^{D1} = \frac{1}{2}(R^M - \theta \pm [(R^M + \theta)^2 + 4\theta[R^M - (\alpha'(A^0 + A^1)A^1 + \alpha(A^0 + A^1))]^{\frac{1}{2}}$$

The negative root actually corresponds to a negative quantity of required equity, so we rule this out. Similarly, we will get

$$R^{D0} = \frac{\gamma(A^0, A^1, M^0) + \sqrt{\gamma(A^0, A^1, M^0)^2 - 4(2-\lambda)\varphi(A^0, A^1, M^0)}}{2(2-\lambda)}$$

Once again the negative root can be ruled out as it implies a negative quantity of equity. By our regularity assumptions, R^{D0} and R^{D1} will exist and be positive. It remains to show however that an R^W exists that satisfies item 6. Note that when $R^W = 0$, and $A^0 = W$ we will have $R^{D0} > R^{D1}$ by (4). Since by items 2 and 3 R^{D0} and R^{D1} are both continuous in R^W there exists and interval $(0, \overline{R^W})$ such that if $R^W \in (0, \overline{R^W})$ we will still have $R^{D0} > R^{D1}$.Consider a $R^{W'}$ from this interval. We will have $R^{D0} > R^{D1}$ for any possible $A^0 < W$. Then choosing an $R^{W*} \leq \min(1, R^{W'})$ will satisfy item 6 for any $A^0 < W$. Since items 1-9 were necessary conditions for an equilibrium, it is clear that this equilibrium specification is unique.

Proof of Corollary 2. We will start with showing that $\frac{\partial R^{D1}}{\partial M^0} < 0$.We have that the first order condition of the banks optimization with respect to $A^1 = D^0 + E^0$ is given by:

$$R^{D1^2} - (R^M - \theta)R^{D1} - R^M - (\alpha'(A^0 + A^1)A^1 + \alpha(A^0 + A^1) = 0 \qquad (17)$$

Implicitly differentiating this with respect to A^1 gives us

$$\frac{\partial R^{D1}}{\partial A^1} = \frac{-\theta[\alpha''(A^0 + A^1)A^1 + 2\alpha'(A^0 + A^1)]}{2R^{D1} - (R^M - \theta)}$$

When $\alpha(.)$ is convex, the numerator of the right hand side is negative. Also when R^{D1} takes the value given in proposition 1, the dominator will be given by

$$[(R^M + \theta)^2 + 4\theta[R^M - (\alpha'(A^0 + A^1)A^1 + \alpha(A^0 + A^1))]^{\frac{1}{2}}$$

which must be greater than zero. Thus $\frac{\partial R^{D1}}{\partial A^1} < 0$. Similarly, differentiating (17) with respect to A^0 yields

$$\frac{\partial R^{D1}}{\partial A^0} = \frac{-\theta[\alpha''(A^0 + A^1)A^1 + \alpha'(A^0 + A^1)]}{2R^{D1} - (R^M - \theta)}$$

Which is negative for the same reason as above. Since $\frac{\partial A^0}{\partial M^0}, \frac{\partial A^1}{\partial M^0} > 0$ we will have $\frac{\partial R^{D0}}{\partial M^0}, \frac{\partial R^{D1}}{\partial M^0} < 0$

The first order condition of the bank's optimization with respect to $A^0 = D^0 + E^0$ can be written as:

$$(2 - \lambda)R^{D0^2} + \gamma R^{D0} + \varphi = 0$$

Differentiating this with respect to A^0 gives us:

$$\frac{\partial R^{D0}}{\partial A^0} = \frac{-(\frac{\partial \gamma}{\partial A^0}R^{D0} + \frac{\partial \varphi}{\partial A^0})}{2(2 - \lambda)R^{D0} + \gamma}$$

Since $\frac{\partial R^{D1}}{\partial A^0}, \frac{\partial R^{D1}}{\partial A^1} < 0$ and $R^{D0} > 0$, we will have that $\frac{\partial \gamma}{\partial A^0}R^{D0} + \frac{\partial \varphi}{\partial A^0} > 0$. Substituting in R^{D0} from proposition 1 makes the denominator equal to $\sqrt{\gamma^2 - 4(2 - \lambda)\varphi}$ which must be positive. So we have that $\frac{\partial R^{D0}}{\partial A^0} < 0$ which implies that $\frac{\partial R^{D0}}{\partial M^0} < 0$. Since R^M is exogenously given, the corollary is then proved.

Proof of Proposition 3. If $R^{FQ} > R^1$, demand for RRPs exceed supply. If $R^{FQ} < R^1$, demand for RRPs is zero and the market does not clear. Neither of these are possible equilibria, hence $R^{FQ} = R^1$ in any equilibrium. This directly implies that households are indifferent between re-investing in the unshocked bank and investing in the MMF at $t = 1$, and therefore market clearing implies the expression for A^1. Then, $A^1 < \lambda(D^0 + E^0)R^W$ implies that $\alpha(D^0 + D^1) < \alpha(\lambda(D^0 + E^0)R^W)$. Item 2 and 3 in proposition 1 implies that the long- and short-term deposit rates both increase relative to the equilibrium without RRPs due to the decrease in A^1, and Item 5 implies that the bond rate increases accordingly..

Proof of Proposition 4. Equality of the rates is immediate from proposition 4. Suppose that $RP^{FR} < RP^{FQ}$, and denote by $A^{1,FR}$ and $A^{1,FQ}$ the corresponding re-deposit volumes. Date $t = 1$ market clearing implies that $A^{1,FR} > A^{1,FQ}$, which in turn implies $R^{1,FR} < R^{1,FQ}$ and hence $R^{1,FR} < R^{FR} = R^{FQ} = R^{1,FQ}$, a contradiction. An analogous argument for $RP^{FR} > RP^{FQ}$ implies that $RP^{FR} = RP^{FQ}$ in any equilibrium. Equality of the $t = 1$ deposit volumes follows from market clearing.

Proof of Proposition 5. First assume λ^L occurs. Suppose $R^{D1} > R^{FR}$. Then there would be no demand for the ON FRFA RRP, and $A^1 = \lambda^L(D^0 + E^0)R^W$. However, since we have that

$$\begin{aligned} & R^{D1} \\ > \; & R^{FR} \\ > \; & \frac{1}{2}(R^M - \theta) + \frac{1}{2}\{(R^M + \theta)^2 - 4[\alpha'(A^0 + \lambda^L A^0 R^W)A^1 + \alpha(A^0 + \lambda^L A^0 R^W)]\}^{\frac{1}{2}} \end{aligned}$$

by assumption, the unshocked bank will not be willing to supply $\lambda^L A^0 R^W$ in $t = 1$ deposits, which is inconsistent with market clearing. Thus, we cannot have $R^{D1} > R^{FR}$. Now suppose that $R^{D1} < R^{FR}$. Then there will be no demand for $t = 1$ bank liabilities $(D^1, E^1 = 0)$. However since

$$R^{FR} = \frac{1}{2}(R^M - \theta) + \frac{1}{2}\{(R^M + \theta)^2 - 4[\alpha'(A^0)A^1 + \alpha(A^0)]\}^{\frac{1}{2}},$$

banks will want to supply positive deposits, which is also inconsistent with market clearing. Therefore, we cannot have $R^{D1} < R^{FR}$. Thus, we have established that $R^{D1} = R^{FR} = R^{D1*}$ when λ^L occurs. Now suppose that λ^H occurs and that $R^{D1} > R^{FR}$. Since $\lambda^H > \lambda^L$, we will have that

$$\begin{aligned} & R^{D1} \\ > \; & R^{FR} \\ > \; & \frac{1}{2}(R^M - \theta) + \frac{1}{2}\{(R^M + \theta)^2 - 4[\alpha'(A^0 + \lambda^H A^0 R^W)A^1 + \alpha(A^0 + \lambda^H A^0 R^W)]\}^{\frac{1}{2}} \end{aligned}$$

by (11). By the same argument as in the previous part, we have that $R^{D1} > R^{FR}$ is a contradiction. The case where $R^{D1} < R^{FR}$ is eliminated by an identical argument as in the previous part. Thus we have that $R^{D1} = R^{FR} = R^{D1*}$ in either state.

Proof of Proposition 6. The fact that $R^{D1} = R^{D1*}$ when λ^H occurs is true by the definition of RP^{FQ^*}. When λ^L occurs and $t = 1$ deposits are demanded in positive amounts, we must have

$$
\begin{aligned}
& R^{D1} = R^{FQ} \\
= \; & \frac{1}{2}(R^M - \theta) + \\
& \frac{1}{2}\{(R^M + \theta)^2 - 4[\alpha'(A^0 + \lambda^L A^0 R^W - RP^{FQ^*})A^1 + \alpha(A^0 + \lambda^L A^0 R^W - RP^{FQ^*})]\}^{\frac{1}{2}} \\
> \; & \frac{1}{2}(R^M - \theta) + \\
& \frac{1}{2}\{(R^M + \theta)^2 - 4[\alpha'(A^0 + \lambda^H A^0 R^W - RP^{FQ^*})A^1 + \alpha(A^0 + \lambda^H A^0 R^W - RP^{FQ^*})]\}^{\frac{1}{2}} \\
= \; & R^{D1*}.
\end{aligned}
$$

Thus, we will have that $R^{D1} = R^{FQ} \geq R^{D1*}$

Proof of Lemma 7. This is immediate as there will be no demand for RRPs when bonds are the more attractive option for the MMF.

Proof of Proposition 8. We define $R^{D0}(RP^{TM})$ to be the equilibrium R^{D0} given in proposition 1 when the central bank offers RP^{TM} term RRPs. Similarly we define $R^{D1}(RP^{TM})$. We define $\widehat{RP^{TM}} = M - \overline{M}$. We let

$$\widehat{R^{TM}} = R^{D0}(\widehat{RP^{TM}})(1 - \frac{\lambda}{2}) + \frac{\lambda}{2}[R^W R^{D1}(\widehat{RP^{TM}}) - (R^W - 1)\theta] \tag{18}$$

Consider a the central bank setting R^{TM} such that $\overline{R^B} < R^{TM} < \widehat{R^{TM}}$. The MMF will demand only RRPs. Since there is excess bond supply the bond rate must rise up to $R^B = R^{TM}$ so that MMFs are willing to hold positive quantities of RRPs and bonds. We also must have households willing to hold deposits for market clearing. Thus D^0, and consequently D^1, will fall, being replaced with MMF share holdings, so that (18) is satisfied by increasing R^{D0} and R^{D1}. This can be accomplished as $R^{D0}(RP^{TM})$ and $R^{D1}(RP^{TM})$ are continuous functions and we can thus apply intermediate value theorem to find the appropriate deposit levels. Since $M - R^{TM} > M - \widehat{R^{TM}} = \overline{M}$ no interbank lending will occur.

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Appendix 3: Figures

Figure 1

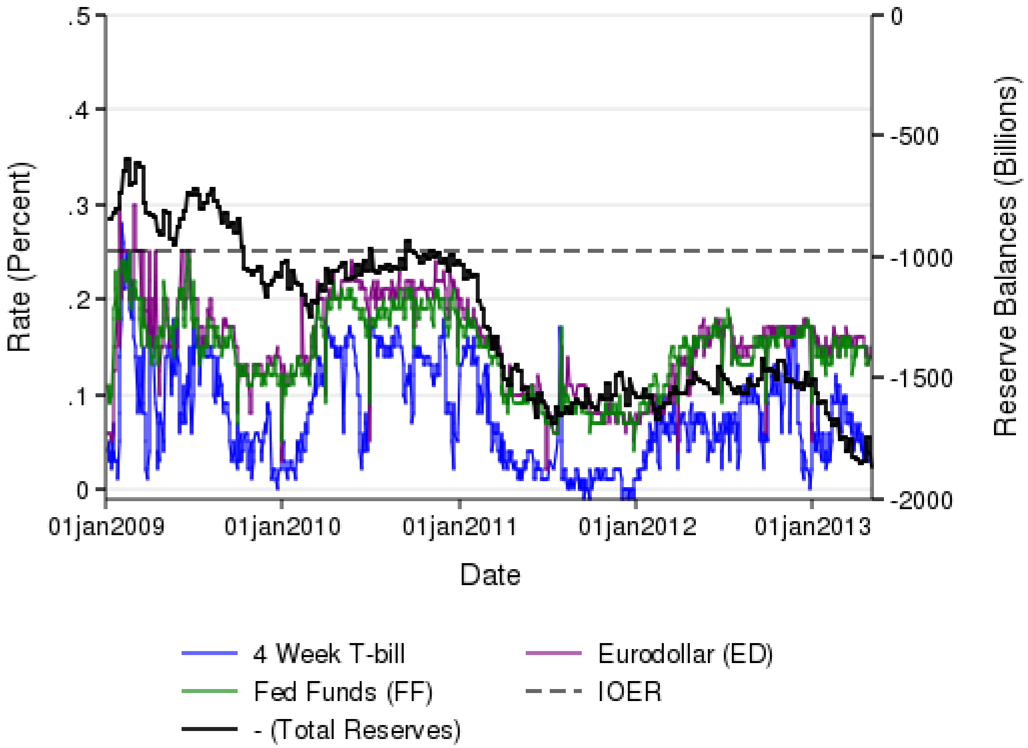
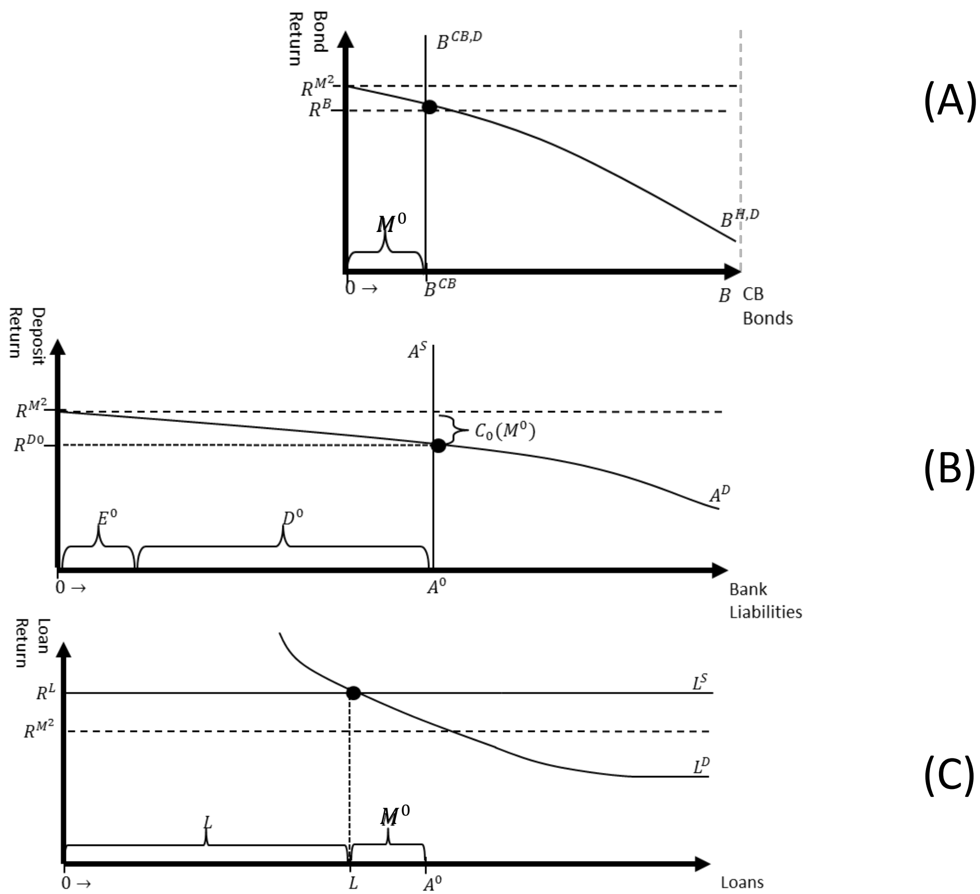


Figure 2

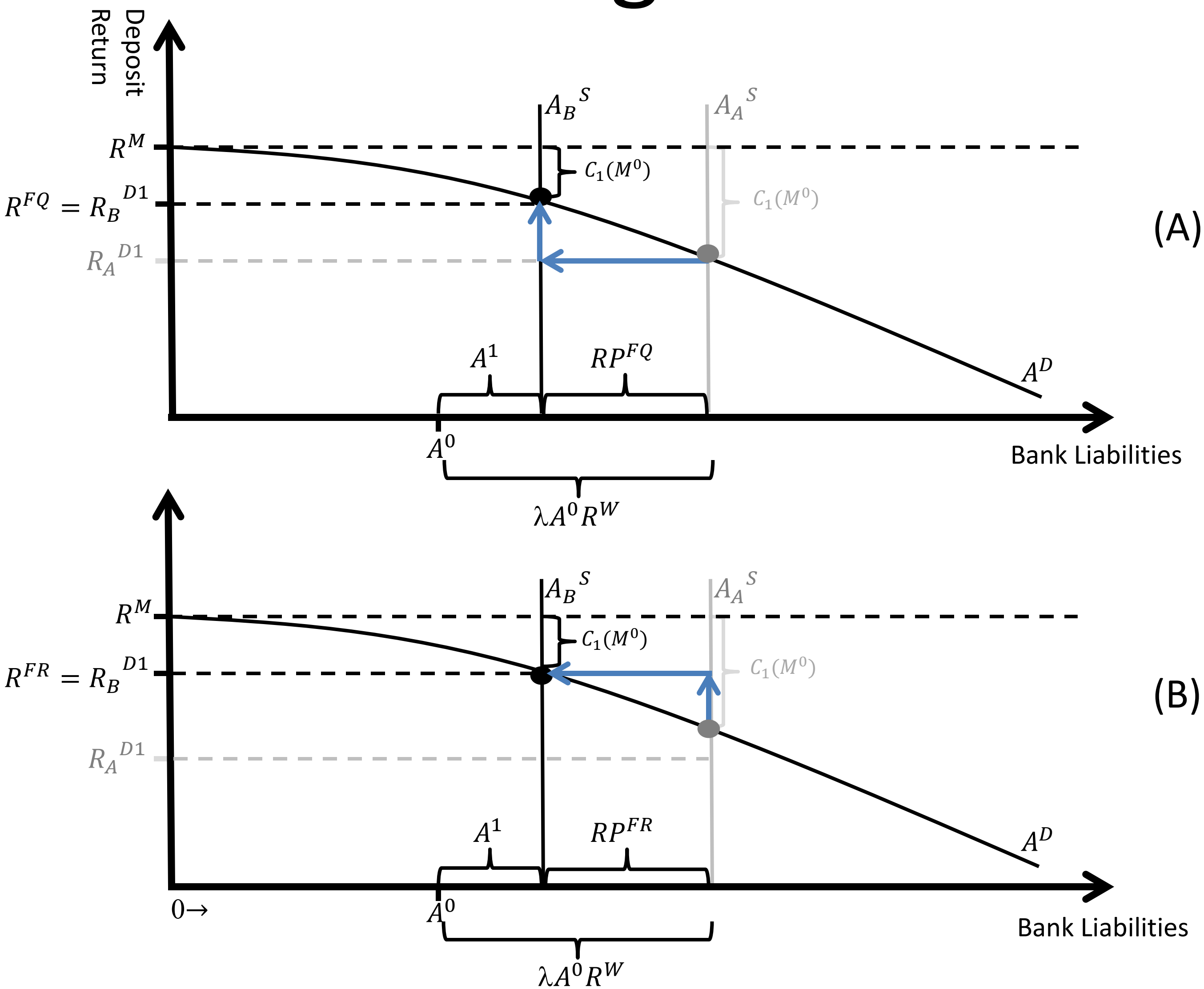


(A) Bond market: Household bond holdings increase leftward. As households hold more bonds, deposits decrease in equilibrium. Deposit rates are less depressed by bank equity requirements, and the equilibrium bond rate rises.

(B) Bank Liabilities: Bond market clearing and the binding resource constraint forces household supply of bank liabilities to be perfectly inelastic. Banks' demand for liabilities are decreasing in the deposit rate.

(C) Loan market: Banks' loan supply correspondence is above IOER representing potential interbank lending costs.

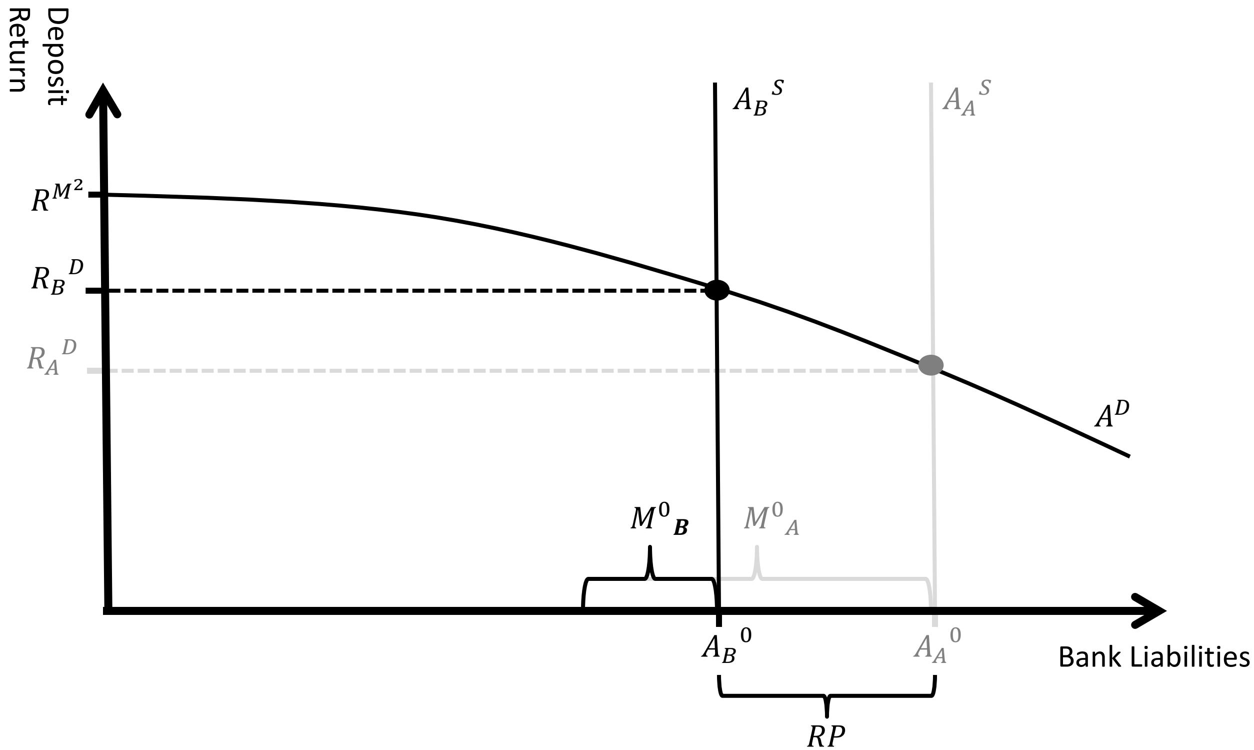
Figure 3



(A) Bank liability market with fixed-quantity RRPs: Liabilities supplied by households mechanically decrease by the amount of RRPs supplied, which forces the deposit rate up to equal the market-determined RRP rate. The presence of RRPs decreases equity requirements, thereby decreasing the spread between the deposit rate and IOR.

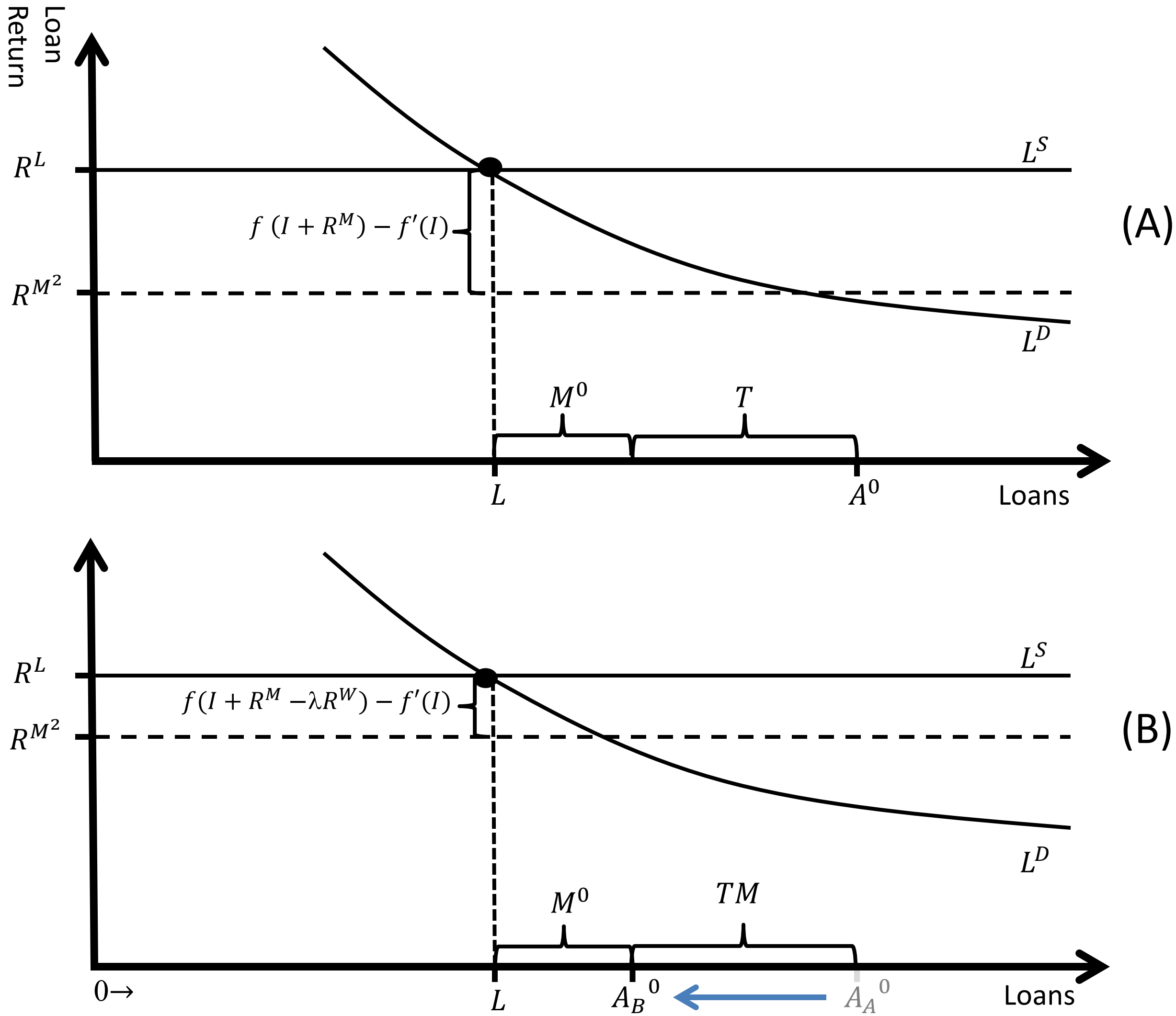
(B) Deposit market with fixed-rate RRPs: The deposit rate increases to match the exogenous RRP rate and market clearing forces household liability supply to decrease. As in (A), smaller equity requirements decrease the spread between the deposit rate and IOR.

Figure 4



Deposit market with daily RRPs: The presence of RRPs forces down the household supply of liabilities one-for-one with reserves. Fewer liabilities imply smaller equity requirements, which decreases the spread between the deposit rate and IOR.

Figure 5



(A) Loan market with TDF: The TDF soaks up reserves from the banks, activating the interbank market and driving a wedge between the loan rate and IOR. Households supply the same quantity of liabilities.

(B) Loan market with term RRPs: The RRPs force down household liability supply one-for-one with reserves. The drop in reserves drives a wedge between the loan rate and IOR, but the drop in liabilities dampens the required volume of interbank lending relative to (A). Hence, the spread between the loan rate and IOR is smaller for term RRPs.